Preambles Based on Power-Split-Ratio for GFRA Systems with Open-Loop Power Control

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***Abstract*—This letter aims to improve the active user detection (AUD) and channel estimation (CE) performance in grant-free random access (GFRA) systems with open-loop power control. In particular, the preamble based on the power-split-ratio (PSR) is designed, where the preamble consists of two sub-preambles generated by multiplying the same sequence with different power coefficients. The ratio of two coefficients is defined as PSR, which can be used to eliminate the effects of uncertain power and detect preamble collisions. Based on the designed preamble, a two-step AUD scheme is proposed to detect the sequence and PSR, followed by the CE scheme. Numerical results show that the proposed PSR-based preamble significantly outperforms existing preambles in GFRA systems with open-loop power control.**

***Index Terms*—Massive machine-type communications, open-loop power control, preamble collision, user detection.**

I. Introduction

In recent years, massive machine-type communications (mMTC) has become a dominant paradigm of communications among Internet of Things (IoT) devices, characterized by massive connections and sporadic transmission of short packets [1]. It poses big challenges for the traditional grant-based random access protocol to accommodate these unique features due to the excessive latency and signaling overhead [2]. As a promising way, the grant-free random access (GFRA) protocol has been proposed to solve the problem by allowing users to transmit packets consisting of the preamble and data without scheduling grant from the base station (BS).

In GFRA systems, without user scheduling, the BS has to perform the active user detection (AUD) and channel estimation (CE) based on the preamble before data detection, where the results of AUD and CE are reliable only if there are no preamble collisions. Because users transmitting the same preamble cannot be distinguished and they will be identified as one active user, resulting in the failure of AUD and a large bias in CE. Several preamble structures have been proposed to reduce preamble collisions by expanding the size of the preamble pool. Taking advantage of the non-orthogonal sequences, preambles based on Gaussian sequences [3] and multiple root Zadoff-Chu (ZC) sequences [4] have been proposed, which are beneficial in reducing preamble collisions. However, they are not qualified for CE when there are numerous users due to severe non-orthogonal interference. On the contrary, orthogonal sequences are favorable for CE, but not for reducing preamble collisions. It has been demonstrated that the AUD and CE performance of a single orthogonal preamble (SOP) is mainly limited by preamble collisions due to the insufficient size of the preamble pool [5]. To address the issue, the concatenated orthogonal preamble (COP) has been proposed in [6], which enlarges the preamble pool by sending multiple randomly selected orthogonal preambles successively. COP has reduced preamble collisions efficiently with lower non-orthogonality, but it only outperforms SOP when the number of antennas is small [7]. In summary, these schemes have reduced preamble collisions with acceptable CE performance.

It is worth noting that all the above schemes are proposed under the assumption of perfect power control, where the received power is proportional to the number of users multiplexed on a preamble. However, there lacks closed-loop power control in GFRA systems, and the received power is randomly distributed over a wide range [1]. Open-loop power control makes the schemes of detecting collisions based on total received power no longer feasible. Therefore, it is necessary to design preambles enabling accurate AUD and CE for GFRA systems with open-loop power control.

In this letter, we, for the first time, propose the preamble based on the power-split-ratio (PSR) to solve the problem, where collisions are detected based on the PSR rather than the received power. Specifically, the proposed preamble is an enhancement to SOP and COP to accommodate the absence of power control. Different from traditional SOP and COP, each orthogonal sequence is divided into two consecutive sub-sequences, which are generated by multiplying the same sequence with different power coefficients. Such two coefficients constitute a coefficient pair, which is randomly selected from a predefined set. The ratio of the two coefficients is defined as PSR, and there exists a bijection between the coefficient pair and PSR. Therefore, for sub-sequences, the ratio of their received power mainly depends on the PSR instead of the uncertain power, which can be used to detect collided users. In detail, the orthogonal sequence detection is first performed. After an orthogonal sequence being detected,

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the BS calculates its PSR and decides whether there is a collision. If the PSR corresponds to a coefficient pair in the pre-defined set, there has been only one user. Otherwise, the detected PSR is the combination of several PSRs, which means that collisions have occurred. Subsequently, based on the AUD results, the transmission power and channel vector can be estimated accordingly. To highlight the novelty of the proposed preamble, the preamble structure of PSR-based SOP is first given, followed by AUD and CE schemes. Since COP is composed of multiple consecutive SOPs, the PSR-based SOP can be easily extended to the PSR-based COP. In addition, simulations are performed to evaluate the performance of PSR-based SOP and PSR-based COP, which validate the superiority of the proposed PSR-based preamble.

The rest of this paper is organized as follows. In Section II, the system model of GFRA system is illustrated. In Section III, the PSR-based SOP is proposed, followed by the AUD and CE scheme. In Section IV, simulation results are presented to evaluate the performance of the proposed scheme. We conclude the paper in Section V.

Notation: The matrices and vectors are denoted by boldface upper and lower case letters, respectively. $\mathbf{I}_M$ is the $M \times M$ unit matrix. The inverse, transpose, and complex conjugate transpose operators are denoted by $(\cdot)^{\dagger}$, $(\cdot)^T$ and $(\cdot)^H$, respectively. $\|\cdot\|$ denotes the Euclidean norm.

II. System Model

Consider a single-cell GFRA system consisting of a BS equipped with M antennas and multiple single-antenna users uniformly distributed in the cell. In each random access slot, it is assumed that there are K active users accessing the channel by sending preambles followed by the data to the BS, where preambles serve as the identification of users to facilitate the BS to perform AUD and CE.

As shown in Fig. 1, $\mathbf{S} = [\mathbf{s}^1, \cdots, \mathbf{s}^j, \cdots, \mathbf{s}^J] \in \mathbb{C}^{1\times N}$ is the transmitted preamble of length N, which is composed of $J \geqslant 1$ consecutive orthogonal sub-preambles. The superscript j denotes the j-th sub-orthogonal preamble. $\mathbf{s}^l$ is randomly selected from the preamble pool $\mathcal{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_l, \cdots, \mathbf{s}_L]^{L\times L}, l = 1, \cdots, L$, where $\mathbf{s}_l \in \mathbb{C}^{1\times L}$ is the single-root Zadoff-Chu (ZC) sequence of length $L = \frac{N}{J}$ with cyclic shift l. When $J = 1$, $\mathbf{S}$ is the SOP, otherwise, it is COP.

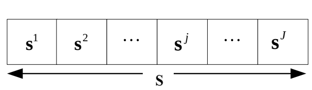

Fig. 1. Traditional preamble structure.

At the BS, the received signal is

$$\mathbf{Y} = \left[\mathbf{Y}^1, \mathbf{Y}^2, \cdots, \mathbf{Y}^j, \cdots, \mathbf{Y}^J\right] \in \mathbb{C}^{M\times N}, \tag{1}$$

where $\mathbf{Y}^j$ is the received signal of j-th sub-preamble, that is,

$$\mathbf{Y}^j = \sum_{k=1}^{K} \sqrt{P_{T,k}}\tilde{\mathbf{h}}_k \mathbf{s}_k^j + \mathbf{W} \in \mathbb{C}^{M\times L}, \tag{2}$$

where $\mathbf{s}_k^j$ is the j-th sub-preamble selected by user k. $P_{T,k}$ is the transmission power of user k, and $\tilde{\mathbf{h}}_k \in \mathbb{C}^{M\times 1}$ is the channel vector between user k and the BS. It is assumed that $\tilde{\mathbf{h}}_k = \sqrt{h_{dk}}\mathbf{h}_k$ with $\sqrt{h_{dk}}$ is the large-scale fading coefficient and $\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I}_{M\times 1})$ is the uncorrelated small-scale fading coefficient, which has been widely used in GFRA systems for analysis and simulations [6], [7]. $\mathbf{W}$ is the circularly symmetric complex Gaussian (CSCG) noise with zero-mean and variance δ^2, and the signal-to-noise ratio (SNR) of user k is $\gamma_k = \frac{P_{T,k}h_{dk}}{\delta^2}$. Without loss of generality, δ^2 is set to 1 [2], [3].

Note that due to the absence of power control, the received power of users $P_k = P_{T,k}h_{dk}$ is different from each other. In this letter, it is assumed that P_k is distributed in the range of $[P_{\text{min}}, P_{\text{max}}]$. Therefore, $\mathbf{Y}^j$ can be rewritten as

$$\mathbf{Y}^j = \sum_{k=1}^{K} \sqrt{P_k}\mathbf{h}_k \mathbf{s}_k^j + \mathbf{W} \in \mathbb{C}^{M\times L}. \tag{3}$$

Based on the received signal $\mathbf{Y}$, the BS performs AUD by detecting preambles selected by active users and also estimates channel vectors to provide information for data detection.

III. Proposed PSR-Based Preamble

In this section, the PSR-based preamble is proposed to improve the detection performance under conditions without power control, which splits each sequence into two consecutive sub-sequences with different power coefficients, thereby eliminating the effect of uncertain power by detecting PSR. For ease of representation, here we take $J = 2$ as an example to illustrate the proposed preamble, which can be extended to $J = 2i, i > 1$ cases easily. The structure of the proposed preamble is presented first, followed by the dedicated AUD and CE scheme.

A. PSR-based preamble

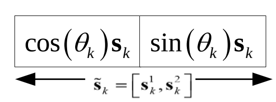

Fig. 2. Proposed PSR-based preamble.

The designed preamble is shown in Fig. 2, the preamble $\tilde{\mathbf{s}}_k$ consists of two sub-preambles generated by multiplying $\mathbf{s}_k$ with different power coefficients $\cos(\theta_k)$ and $\sin(\theta_k)$, i.e.,

$$\tilde{\mathbf{s}}_k = [\cos(\theta_k)\mathbf{s}_k, \sin(\theta_k)\mathbf{s}_k] \in \mathbb{C}^{1\times N}, \tag{4}$$

where $\mathbf{s}_k$ is the orthogonal sequence of length $\frac{N}{2}$. Define $r_k = \frac{\sin^2(\theta_k)}{\cos^2(\theta_k)}$ as the PSR. Therefore, (4) can also be expressed as

$$\tilde{\mathbf{s}}_k = \left[\sqrt{\frac{1}{1+r_k}}\mathbf{s}_k, \sqrt{\frac{r_k}{1+r_k}}\mathbf{s}_k\right] \in \mathbb{C}^{1\times N}, \tag{5}$$

where r_k is randomly selected from $\mathcal{R} = \{r_a, r_b\}$ with $r_a \leqslant 1$ and $r_b > 1$. Although the length of $\mathbf{s}_k$ is shorten by half, with the randomly selected r_k, the size of the preamble pool of PSR-based preamble still equals to N.

Note that the main purpose of this work is to reduce the impact of the power uncertainty instead of preamble collisions, therefore, the size of $\mathcal{R}$ is set to 2 so that the size of preamble

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pool and the probability of no collision P_{nc} remain the same as SOP of length N, which means that the performance improvement is caused only by detecting $\tilde{\mathbf{s}}_k$ under the same P_{nc}. As for problems caused by preamble collisions, they can be solved by increasing J. Taking $J = 4$ (i.e., $i = 2$) as an example, the PSR-based preamble is extended from (5) to

$$\tilde{\mathbf{s}}_k = \left[\sqrt{\frac{1}{1+r_{k_1}}}\mathbf{s}_k^1, \sqrt{\frac{r_{k_1}}{1+r_{k_1}}}\mathbf{s}_k^1, \sqrt{\frac{1}{1+r_{k_2}}}\mathbf{s}_k^2, \sqrt{\frac{r_{k_2}}{1+r_{k_2}}}\mathbf{s}_k^2\right], \tag{6}$$

where $\mathbf{s}_k^1$ and $\mathbf{s}_k^2$ are orthogonal sequences of length $\frac{N}{4}$, which are randomly selected from the preamble pool $\mathcal{S}$. r_{k_1} and r_{k_2} are randomly selected from the PSR set $\mathcal{R}$. In this case, the size of preamble pool is enlarged to $\left(\frac{N}{2}\right)^2$, and the probability of collisions can be reduced accordingly. In summary, the size of the PSR-based preamble pool is $\left(\frac{N}{i}\right)^i$, and P_{nc} is

$$P_{\text{nc}} = \left(1 - \left(\frac{i}{N}\right)^i\right)^{K-1}, i \geqslant 1. \tag{7}$$

Substituting (5) into (3), the received signal is expressed as

$$\begin{aligned}
\mathbf{Y} &= \sum_{k=1}^{K} \sqrt{P_k}\mathbf{h}_k\tilde{\mathbf{s}}_k + \mathbf{W} = \left[\mathbf{Y}^1, \mathbf{Y}^2\right] \in \mathbb{C}^{M\times N},\\
\mathbf{Y}^1 &= \sum_{k=1}^{K} \sqrt{\frac{P_k}{1+r_k}}\mathbf{h}_k\mathbf{s}_k + \mathbf{W} \in \mathbb{C}^{M\times \frac{N}{2}},\\
\mathbf{Y}^2 &= \sum_{k=1}^{K} \sqrt{\frac{P_k r_k}{1+r_k}}\mathbf{h}_k\mathbf{s}_k + \mathbf{W} \in \mathbb{C}^{M\times \frac{N}{2}}.
\end{aligned} \tag{8}$$

B. Preamble detection

After receiving $\mathbf{Y}$ from the uplink transmission, the BS detects PSR-based preambles to identify active users. Since $\tilde{\mathbf{s}}_k$ is identified by $\mathbf{s}_k$ and r_k, AUD is performed in two steps, where the orthogonal sequence detection is first conducted to find out the selected orthogonal sequences, and then PSR is detected to determine the presence of preamble collisions.

In view of the advantageous autocorrelation and cross-correlation properties of the single-root ZC sequence, the sequence $\mathbf{s}_k$ with $k = 1, \cdots, K$ can be detected by calculating the correlations of the received signal and $\mathbf{s}_k^H$. Denote the cases that $\mathbf{s}_k$ has not been selected and has been selected as $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Therefore, we have

$$\mathbf{Y}^1\mathbf{s}_k^H \big| \mathcal{H}_0 = \mathbf{W}\mathbf{s}_k^H, \ \ \mathbf{Y}^2\mathbf{s}_k^H \big| \mathcal{H}_0 = \mathbf{W}\mathbf{s}_k^H. \tag{9}$$

$$\begin{aligned}
\mathbf{Y}^1\mathbf{s}_k^H \big| \mathcal{H}_1 &= \frac{N}{2}\sum_{k\in\mathcal{U}_k} \sqrt{\frac{P_k}{1+r_k}}\mathbf{h}_k + \mathbf{W}\mathbf{s}_k^H,\\
\mathbf{Y}^2\mathbf{s}_k^H \big| \mathcal{H}_1 &= \frac{N}{2}\sum_{k\in\mathcal{U}_k} \sqrt{\frac{P_k r_k}{1+r_k}}\mathbf{h}_k + \mathbf{W}\mathbf{s}_k^H,
\end{aligned} \tag{10}$$

where $\mathcal{U}_k$ denotes the set of users which have selected $\mathbf{s}_k$ as the preamble. It is obviously that $\mathbf{s}_k$ can be detected by calculating $\left\|\mathbf{Y}^1\mathbf{s}_k^H\right\|^2$ and $\left\|\mathbf{Y}^2\mathbf{s}_k^H\right\|^2$. In addition, $\frac{\left\|\mathbf{Y}^2\mathbf{s}_k^H\right\|^2}{\left\|\mathbf{Y}^1\mathbf{s}_k^H\right\|^2}$ mainly depends on r_k rather than P_k, which can be used to combat the power uncertainty. Therefore, the test statistics of orthogonal sequence detection and PSR detection are given as follows.

1) Orthogonal sequence detection: The test statistic of orthogonal sequence detection is defined as

$$T_{\text{osd}} = \frac{1}{M}\frac{2}{N}\left\|\mathbf{Y}^1\mathbf{s}_k^H\right\|^2 + \frac{1}{M}\frac{2}{N}\left\|\mathbf{Y}^2\mathbf{s}_k^H\right\|^2. \tag{11}$$

For ease of representation, $\frac{1}{M}\frac{2}{N}\left\|\mathbf{Y}^1\mathbf{s}_k^H\right\|^2$ and $\frac{1}{M}\frac{2}{N}\left\|\mathbf{Y}^2\mathbf{s}_k^H\right\|^2$ are denoted as G_1 and G_2, respectively.

G_1 and G_2 are random variables that obey the Chi-square distribution, and according to the central limit theorem (CLT), they can be approximated by Gaussian distributions when MN is large [8], [9], that is

$$\begin{aligned}
&G_1|\mathcal{H}_0 \sim \mathcal{N}\left(1, \frac{1}{M}\right), \ G_2|\mathcal{H}_0 \sim \mathcal{N}\left(1, \frac{1}{M}\right),\\
&G_1|\mathcal{H}_1 \sim \mathcal{N}\left(\frac{NP_k}{2(1+r_k)} + 1, \frac{NP_k}{M(1+r_k)} + \frac{1}{M}\right),\\
&G_2|\mathcal{H}_1 \sim \mathcal{N}\left(\frac{NP_k r_k}{2(1+r_k)} + 1, \frac{NP_k r_k}{M(1+r_k)} + \frac{1}{M}\right).
\end{aligned} \tag{12}$$

Note that the purpose of this step is to detect the presence of $\mathbf{s}_k$, not figure out the number of users colliding in choosing $\mathbf{s}_k$. In other words, as long as one user has selected $\mathbf{s}_k$, hypothesis H_1 holds. Therefore, the distribution of $G_1|\mathcal{H}_1$ and $G_2|\mathcal{H}_1$ only consider the case of one user.

Base on (12), the distribution of T_{osd} is

$$T_{\text{osd}}\big|\,\mathcal{H}_0 \sim \mathcal{N}\left(2, \frac{2}{M}\right), T_{\text{osd}}\big|\,\mathcal{H}_1 \sim \mathcal{N}\left(\frac{NP_k}{2} + 2, \frac{NP_k}{M} + \frac{2}{M}\right). \tag{13}$$

Hence, $\mathbf{s}_k$ can be detected based on the following decision rule,

$$\begin{cases} \mathcal{H}_0, & T_{\text{osd}} < \lambda_s \\ \mathcal{H}_1, & T_{\text{osd}} \geqslant \lambda_s, \end{cases} \tag{14}$$

where λ_s is the threshold that satisfies the condition of $f_{T_{\text{osd}}|\mathcal{H}_0}(\lambda_s) = f_{T_{\text{osd}}|\mathcal{H}_1}(\lambda_s)$. $f_{T_{\text{osd}}|\mathcal{H}_0}(\cdot)$ and $f_{T_{\text{osd}}|\mathcal{H}_1}(\cdot)$ are the probability density function (PDF) of $T_{\text{osd}}\big|\,\mathcal{H}_0$ and $T_{\text{osd}}\big|\,\mathcal{H}_1$, which are given as (13).

C. PSR detection

Once $\mathbf{s}_k$ has been detected, the BS detects the corresponding r_k to determine whether there are collisions in $\mathbf{s}_k$. According to (12), the test statistics of PSR detection is given as

$$T_{\text{psr}} = \frac{G_2 - 1}{G_1 - 1}. \tag{15}$$

For ease of representation, $G_1 - 1$ and $G_2 - 1$ are denoted as $\tilde{G}_1$ and $\tilde{G}_2$, respectively. Hence, the conditional PDF of T_{psr} is

$$\begin{aligned}
f_{T_{\text{psr}}|r_k}(t) &= \int_0^{+\infty} \tilde{g}_1 f_{\tilde{g}_2}(t\tilde{g}_1) f_{\tilde{g}_1}(\tilde{g}_1)\, d\tilde{g}_1\\
&= \frac{1}{r_k\sqrt{\pi}}\frac{\rho}{\text{erf}(q)}\frac{1+t}{\left(1+\frac{t^2}{r_k}\right)^{\frac{3}{2}}}\exp\left\{-\frac{\rho^2(t-r_k)^2}{r_k(r_k+t^2)}\right\},
\end{aligned} \tag{16}$$

with

$$\rho = \frac{\sqrt{M}}{2\sqrt{2}}\sqrt{\frac{NP_k r_k}{1+r_k}}, q = \frac{\sqrt{M}}{2\sqrt{2}}\sqrt{\frac{NP_k}{1+r_k}}. \tag{17}$$

Note that (16) is not normally distributed, however, it has been demonstrated that when the coefficients of variation are smaller than 0.1, the ratio distribution of two independent normal variables can be approximated as the normal distribution [10]. Since $\frac{\delta_1}{\mu_1} \ll 0.1$ and $\frac{\delta_2}{\mu_2} \ll 0.1$ in the case of large M and N, where μ_1 and μ_2 are the means of $\tilde{G}_1$ and $\tilde{G}_2$, respectively. δ_1 and δ_2 are the corresponding standard deviations. Therefore, (16) can be approximated by the following distribution.

$$T_{\text{psr}}|r_k \sim \mathcal{N}\left(r_k, \frac{4r_k\left(r_k+1\right)^2}{MNP_k}\right). \tag{18}$$

Fig. 3 gives a comparison of the estimated PDF of $T_{\text{psr}}|r_k$ with the accurate distribution given in (16), where $M = 50$, $N = 64$ and SNR $= 0$ dB. It is shown that the estimated results accord with the accurate ones, which verifies the reliability of the estimates. In addition, it is obvious that the distribution of T_{psr} with different PSRs can be distinguished easily. Therefore, if $\mathbf{s}_k$ has been selected by one user, the detected T_{psr} is close to r_a or r_b. However, in the case of preamble collision, if $\mathbf{s}_k$ has been sent by two users with different PSRs[1], T_{psr} becomes

$$T_{\text{psr}} = \frac{\frac{r_aP_1}{1+r_a}+\frac{r_bP_2}{1+r_b}}{\frac{P_1}{1+r_a}+\frac{P_2}{1+r_b}} = r_a + \frac{\left(r_b-r_a\right)\left(1+r_a\right)P_2}{P_1\left(1+r_b\right)+P_2\left(1+r_a\right)}. \tag{19}$$

Therefore, in the presence of collisions, T_{psr} is in the range of (r_a, r_b) and it is significantly different from r_a and r_b in general. Note that if P_1 and P_2 are distributed in a particularly wide range, T_{psr} is probably extremely close to r_a or r_b, which means that $T'_{\text{psr}} = \frac{(r_b-r_a)(1+r_a)P_2}{P_1(1+r_b)+P_2(1+r_a)} \approx 0$ or $T''_{\text{psr}} = \frac{(1+r_a)P_2}{P_1(1+r_b)+P_2(1+r_a)} \approx 1$. In other words, when the difference between P_1 and P_2 is so large that the effects of PSRs on T'_{psr} and T''_{psr} is negligible, the power-split ratio of the user with low power cannot be detected, which results in the failure of collision detection.

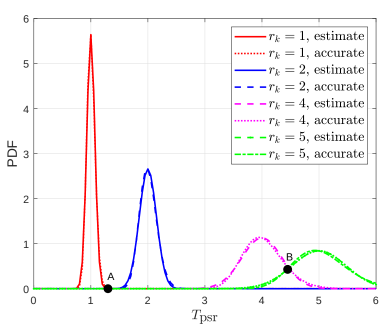

Fig. 3. Distribution of T_{psr}.

In summary, the decision rule of PSR detection is given as

- If $T_{\text{psr}} < \min(\tau_1, \eta_1)$, r_a has been selected, where τ_1 and η_1 are larger than r_a, and they satisfy the conditions of $f_{T_{\text{psr}}|r_a}(\tau_1) = 0$ and $f_{T_{\text{psr}}|r_a}(\eta_1) = f_{T_{\text{psr}}|r_a+1}(\eta_1)$.
- If $T_{\text{psr}} < \max(\tau_2, \eta_2)$, r_b has been selected, where τ_2 and η_2 are smaller than r_b, and they satisfy the conditions of $f_{T_{\text{psr}}|r_b}(\tau_2) = 0$ and $f_{T_{\text{psr}}|r_b}(\eta_2) = f_{T_{\text{psr}}|r_b-1}(\eta_2)$.
- Otherwise, both r_a and r_b have been selected. There are preamble collisions in sending $\mathbf{s}_k$.

For example, when $r_a = 1$ and $r_b = 5$, if $T_{\text{psr}} < 1.3$ or $T_{\text{psr}} > 4.45$, it is declared that $r = 1$ or $r = 5$, respectively. Otherwise, the detected PSR is in the range of $[1.3, 4.45]$, which indicates that both $r_a = 1$ and $r_b = 5$ have been selected.

D. Channel estimation

With the detected $\mathbf{s}_k$ and r_k, the transmission power and channel vector can be estimated accordingly. In detail,

- If there is no preamble collision in $\mathbf{s}_k$, the received signal related to $\mathbf{s}_k$ is

$$\mathbf{Y} = \sqrt{P}\mathbf{h}\tilde{\mathbf{s}}_k + \mathbf{W}, \tag{20}$$

where $\tilde{\mathbf{s}}_k = \left[\sqrt{\frac{1}{1+r_k}}\mathbf{s}_k, \sqrt{\frac{r_k}{1+r_k}}\mathbf{s}_k\right]$. P is estimated based on G_1 and G_2, and the channel vector can be estimated accordingly. That is,

$$\hat{P} = \frac{2\left(G_1+G_2-2\right)}{K}, \ \hat{\mathbf{h}} = \frac{1}{\sqrt{\hat{P}}}\mathbf{Y}\tilde{\mathbf{s}}_k^{\dagger}. \tag{21}$$

- If there are preamble collisions in $\mathbf{s}_k$ and both r_a and r_b have been selected, the received signal related to $\mathbf{s}_k$ is

$$\begin{aligned}\mathbf{Y} &= \sqrt{P_1}\mathbf{h}_1\tilde{\mathbf{s}}_{k_1} + \sqrt{P_2}\mathbf{h}_2\tilde{\mathbf{s}}_{k_2} + \mathbf{W} \\ &= \left[\mathbf{h}_1, \mathbf{h}_2\right]\begin{bmatrix}\sqrt{\frac{P_1}{1+r_a}}\mathbf{s}_k & \sqrt{\frac{P_1r_a}{1+r_a}}\mathbf{s}_k \\ \sqrt{\frac{P_2}{1+r_b}}\mathbf{s}_k & \sqrt{\frac{P_2r_b}{1+r_b}}\mathbf{s}_k\end{bmatrix} + \mathbf{W}.\end{aligned} \tag{22}$$

Here, the mean of G_1 and G_2 are $\frac{K}{2}\left(\frac{P_1}{1+r_a}+\frac{P_2}{1+r_b}\right)+1$ and $\frac{K}{2}\left(\frac{P_1r_a}{1+r_a}+\frac{P_2r_b}{1+r_b}\right)+1$, respectively. Therefore,

$$\begin{aligned}\hat{P}_1 &= \frac{2\left(1+r_a\right)\left(r_bG_1-G_2-2\right)}{K\left(r_b-r_a\right)}, \\ \hat{P}_2 &= \frac{2\left(1+r_b\right)\left(G_2-r_aG_1-2\right)}{K\left(r_b-r_a\right)}.\end{aligned} \tag{23}$$

Then, the channel estimation result is

$$\left[\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2\right] = \mathbf{Y}\begin{bmatrix}\sqrt{\frac{\hat{P}_1}{1+r_a}}\mathbf{s}_k & \sqrt{\frac{\hat{P}_1r_a}{1+r_a}}\mathbf{s}_k \\ \sqrt{\frac{\hat{P}_2}{1+r_b}}\mathbf{s}_k & \sqrt{\frac{\hat{P}_2r_b}{1+r_b}}\mathbf{s}_k\end{bmatrix}^{\dagger}. \tag{24}$$

IV. Numerical Results

In the section, we conduct simulations to evaluate the performance of the proposed PSR-based preambles, where simulation parameters are set to $M = 50$ and $N = 64$. The superiority of PSR-based preambles is verified by comparing with existing preambles including SOP [5] and COP [6].

The comparison of AUD performance versus K is shown in Fig. 4(a), where $r_a = 1$, $r_b = 5$ and SNR is uniformly distributed in $[-5, 5]$ dB. It is obviously that the PSR-based preambles outperform than traditional preambles, and the superiority becomes more significant as K increases. In addition, when $J = 4$, the size of preamble pool has been greatly

[1]When collisions have occured, it is mostly likely that a preamble has been selected by two users. For example, when $K = 64$ and $N = 80$, the probability of collision is 0.7133, but the probability that a preamble has been selected by three users is only 0.07. Therefore, when it comes to collisions, we only consider the case that a preamble has been chosen by two users.

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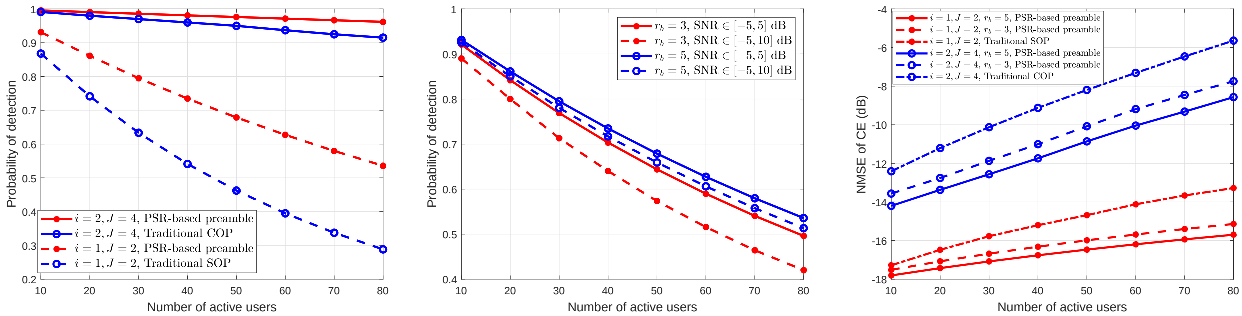

(a) (b) (c)

Fig. 4. Numerical Results: (a) Comparison of AUD performance, $r_b = 5$ and SNR $\in [-5, 5]$ dB. (b) Comparison of AUD performance under different SNR ranges and different r_b, $r_a = 1$. (c) Comparison of CE performance, $r_a = 1$ and SNR $\in [-5, 10]$ dB.

enlarged, which reduces preamble collisions. Therefore, the performance improvement of $J = 4$ is smaller than that of $J = 2$, and the performance gap between PSR-based preamble and COP decreases as J increases.

Fig. 4(b) presents the performance of PSR-based SOP versus SNR ranges and PSRs. It is shown that a wider SNR range leads to the performance degradation. Because with a wider SNR range, collided users are more likely to have large power difference, and the lower power has almost little effect on T_{psr}. In such cases, the collisions cannot be detected. But the performance degradation can be alleviated by enlarging $r_b - r_a$, so that the influence of lower power users can be improved if they have selected a larger r_b, which happens with the probability 0.5. Accordingly, as shown in Fig. 4(b), the performance is less affected by SNR ranges when $r_b = 5$.

Fig. 4(c) shows the normalized mean squared error (NMSE) of CE versus K, which has a significant impact on the data recovery and successful access. Specifically, the NMSE is

$$\text{NMSE} = \frac{1}{E}\sum_{e=1}^{I}\frac{\left\|\hat{\mathbf{h}}_{k,e} - \mathbf{h}_{k,e}\right\|^2}{\left\|\mathbf{h}_{k,e}\right\|^2}, \tag{25}$$

where $\hat{\mathbf{h}}_{k,e}$ and $\mathbf{h}_{k,e}$ are the estimated and accurate channel vectors of user k in e_{th} Monte Carlo simulation. E is the total number of Monte Carlo simulations. Among these schemes, the PSR-based preamble with $r_b = 5$ performs best, since a larger r_b is beneficial for collision detection and power estimation and thus improving the CE performance.

V. Conclusion

In this letter, the PSR-based preamble is proposed to improve the AUD and CE performance in GFRA systems without power control. The effect of power uncertainty is eliminated by PSR so that preamble collisions can be detected through PSR instead of the received power, which enables high-performance detection in the absence of power control. The idea of PSR-based preambles can be applied to existing SOP and COP, and the superiority has been validated by simulation results.

Although PSR greatly reduces the impact of power uncertainty, there are still some issues that deserve further research. Notably, the proposed scheme is feasible when two users are in collision, but the performance degrades in the case of a wider SNR range or collisions with more users. Despite that the performance degradation can be compensated by increasing $|r_b - r_a|$ or introducing more PSRs, the problems have not been thoroughly and extensively addressed. Therefore, there is still a long way to go in designing preambles to support massive connectivity with high power uncertainty. Furthermore, the application of PSR-based preambles in diverse scenarios needs further research to cope with new challenges posed by different transmission characteristics.

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