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A Simple Method for the Performance Analysis of Fluid Antenna Systems under Correlated Nakagami-m Fading

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***Abstract*—By recognizing the tremendous flexibility of the emerging fluid antenna system (FAS), which allows dynamic reconfigurability of the location of the antenna within a given space, this paper investigates the performance of a single-antenna FAS over spatially correlated Nakagami-m fading channels. Specifically, simple and highly accurate closed-form approximations for the cumulative density function of the FAS channel and the outage probability of the proposed system are obtained by employing a novel asymptotic matching method, which is an improved version of the well-known moment matching. With this method, the outage probability can be computed simply without incurring complex multi-fold integrals, thus requiring negligible computational effort. Finally, the accuracy of the proposed approximations is validated, and it is shown that the FAS can meet or even exceed the performance attained by the conventional maximal ratio combining (MRC) technique.**

***Index Terms*—Asymptotic matching, fluid antenna system, nakagami-m fading, spatial correlation, outage probability.**

I. Introduction

The fifth-generation (5G) of wireless mobile networks have recently been deployed worldwide, so industry and academia have already started the race to define the shape of the future sixth-generation (6G). Very recently, a technology that has been gaining momentum is the fluid antenna system (FAS), which is a new paradigm of antenna systems where antennas are equipped with software-controllable fluid structure (e.g., Eutectic Gallium-Indium, Mercury, Galinstan, etc.) that allows dynamic reconfigurability of its position and shape within a given space[1]. Particularly, FAS may help overcome the practical limitations of using multiple antennas in size-constrained devices, and the cost of radio frequency (RF) chains [2, 3]. The fundamental single fluid antenna is built of one RF chain and N fixed locations, so-called ports, distributed in a linear space. Unlike conventional spatial diversity techniques (e.g., maximal ratio combining (MRC)), FAS allows an antenna to freely switch its position among the ports to obtain a more robust channel gain or lower interference, thus providing remarkable gains in diversity, multiplexing, and interference-free communications [4].

A plethora of works have been recently focused on investigating the performance of FAS in wireless communications, where metrics such as ergodic capacity and outage probability (OP) metrics have been investigated in different settings. For instance, in [5], Wong et al. demonstrated that a single-antenna FAS outperforms the traditional MRC in terms of the OP when the number of ports at the fluid antenna is large enough. Also, in [2, 6], Wong et al. studied the achievable performance of FAS in arbitrarily correlated Rayleigh fading channels. Khammassi et al. proposed an approximate expression for the FAS relative channel distribution in [7], where a two-stage approach was proposed. The first phase reduces the number of multi-fold integrals of the OP, while the second represents the OP in a single-integral form by assuming correlated Rayleigh fading channels. Tlebaldiyeva et al. considered a more general small-scale fading channel model on the FAS, in [8], where the OP was found in a single-integral form for a single-antenna N-port FAS over spatially Nakagami-m channel. In [9], by taking advantage of stochastic geometry tools, Skouroumounis and Krikidis derived a closed-form expression for the OP in fluid antenna for large-scale cellular networks. Moreover, Ghadi et al. derived a closed-form formulation of the OP performance in [10] by adopting copula theory to characterize the correlation model (e.g., Frank, Clayton, and Gumbel) between fading channel coefficients. Very recently, the OP behavior of FAS-aided Terahertz communication networks under correlated α-μ fading channels for non-diversity and diversity FAS receivers was investigated by Tlebaldiyeva et al. in [11]. Therein, as in [8], the OP was derived in single-integral expression due to the mathematical intractability of both the α-μ channel model and the diversity FAS underlying system.

Based on the above considerations and motivated by the potential of FAS to provide diversity and remarkable capacity benefits for forthcoming networks, this work exploits the advantages of a novel asymptotic matching method to approximate the FAS's channel distribution into a single approach. Despite the FAS system's intricacy, the authors aim to provide analytically tractable expressions for the outage metric without incurring the prohibitive complexity of special functions or multi/single-fold integrals, which have already been used in previous works. Specifically, a FAS that experiences correlated

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[1]Interested readers can refer to [1] for information on fluid antenna prototypes.

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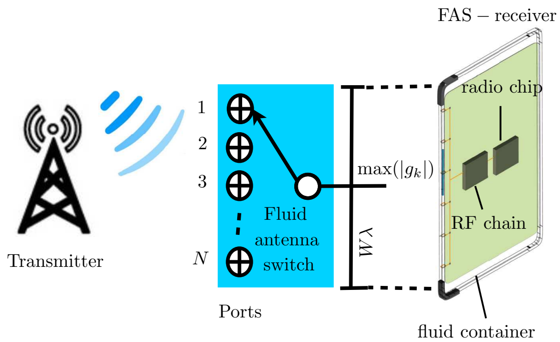

Fig. 1. System Model for single-antenna N-port FAS.

Nakagami-m fading channels is considered, where we propose to approximate the equivalent cumulative density function (CDF) of the FAS by a simple Gamma distribution, where the fitting parameters are estimated via the asymptotic matching method, proposed in [12]. With the CDF of FAS at hand, we derive a simple and highly accurate closed-form expression of the OP valid to the practical correlation models introduced in [6]. It is worth mentioning that such a method outperforms those approaches based solely on the moment matching method (MoM)[2] and improves the computational complexity of cumbersome traditional exact formulations. Compared to the latter, obtained in terms of the correlated join probability density function (PDF)[3], our approach presents a simple form of the OP in terms of well-known functions in the communication theory community, facilitating its numerical evaluation in any computer software. Finally, useful insights into the impact of propagation conditions and the number of ports over the OP performance are also provided.

II. System and Channel Models

We consider a FAS consisting of a single-antenna transmitter (Tx) communicating with a fluid single-antenna that can move freely along N ports equally distributed on a linear space of length $W\lambda$, where λ is the wavelength, and W is the antenna size, as illustrated in Fig. 1. The FAS contains one RF chain, thus only one port can be activated for communication, so the received signal at the k-th port can be defined as

$$Y_k = h_k X + Z_k, \tag{1}$$

with h_k being modeled as a correlated Nakagami-m fading channel since antenna ports are located close to each other. Also, X is the information signal, and Z_k is the additive white Gaussian noise (AWGN) at every port. We assume that FAS can switch the port with the strongest signal for communication, which can be expressed as

$$g_{\text{FAS}} = \max\left(|g_1|, |g_2|, \cdots, |g_k|\right), \ \ \textit{for} \ \ k \in \{1, 2, \cdots, N\}, \tag{2}$$

[2]As stated in [13], some performance metrics in communications systems, such as the OP or bit error rate, are dominated by the channel asymptote at medium to high signal-noise-to-ratio (SNR). So, the asymptotic matching method, compared to the MoM, provides an excellent fit in medium to high SNR, which is a crucial regime in practice. Conversely, the MoM delivers good performance at low SNR, a range of little importance in practical applications.

[3]This solution becomes tedious as the number of ports increases.

where $g_i = |h_i|^2$ for $i \in \{1, \cdots, N\}$ denotes the channel gain of each port in the FAS. Hence, the received SNR, for the FAS can be expressed by

$$\gamma = \frac{P\left|g_{\text{FAS}}\right|}{N_0} = \overline{\gamma}\left|h_{\text{FAS}}\right|, \tag{3}$$

where $\overline{\gamma} = \frac{P}{N_0}$ is the average transmit SNR, with P being the transmit power and N_0 the noise power. Considering this, we aim to provide an approximate statistical model for g_{FAS}, which can be used to obtain the OP distribution straightforwardly.

III. Performance Analysis

In this section, we consider the OP to evaluate the performance of the FAS, which is defined as the probability that the SNR γ is less than a threshold rate, γ_{th}. Therefore, from [8, Eq. (10)], the OP formulation over correlated Nakagami-m random variables (RVs) can be formulated as

$$\begin{aligned}P_{\text{out}}(\gamma_{th}) = &\frac{2^m m}{\Gamma(m)\Omega_1^{2m}}\int_0^{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}}} r_1^{2m}\exp\left(-\frac{m r_1^2}{\Omega_1^2}\right)\\ &\times\prod_{k=2}^{N}\left(1 - Q_m\left(\sqrt{\frac{2m\mu_k^2 r_1^2}{\Omega_1^2\left(1-\mu_k^2\right)}}, \sqrt{\frac{2m\gamma_{th}}{\Omega_k^2\left(1-\mu_k^2\right)\overline{\gamma}}}\right) dr_1,\end{aligned} \tag{4}$$

where $\Gamma(\cdot)$ is the Gamma function, $Q_m(\cdot,\cdot)$ denotes the m-order Marcum Q-function, m is the severity fading parameter, and Ω_k^2 stands for the average channel power of Nakagami-m distribution. Furthermore, the correlation coefficient, denoted by, μ_k, can be defined as [8, Eq. (2)] when the $(N-1)$ ports are referenced to the first port or can be expressed as [6]

$$\mu^2 = \left|\frac{2}{N(N-1)}\sum_{k=1}^{N-1}(N-k)J_0\left(\frac{2\pi k W}{N-1}\right)\right|, \textit{for } \mu_k = \mu \, \forall k, \tag{5}$$

where it is assumed that all the ports do not have a reference port or any port is a reference to any other port, and $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind. It is worth highlighting that as the number of ports increases, the exact solution in (4) becomes more costly, prone to convergence and instability problems, or even impracticable. To circumvent the referred limitation of the exact OP, an approximation for (4) is proposed by using the asymptotic matching method [12], as stated in the following proposition.

Proposition 1. *An approximate expression for the OP of a FAS undergoing correlated Nakagami-m RVs can be obtained as*

$$P_{\text{out}}(\gamma_{th}) \approx \frac{\Upsilon\left(\alpha, \frac{\gamma_{th}}{\beta\overline{\gamma}}\right)}{\Gamma(\alpha)}, \tag{6}$$

where is $\Upsilon(\cdot,\cdot)$, is the lower incomplete gamma function [14, Eq. (6.5.2)] and

$$\alpha = mN, \quad \beta = \left(\frac{1}{\Gamma(\alpha) a_0 \alpha}\right)^{1/\alpha}, \tag{7a}$$

$$a_0 = \frac{m^{m-1}}{\Gamma(m)\Omega_1^{2m} m!^{N-1}}\prod_{k=2}^{N}\left(\frac{m}{\Omega_k^2\left(1-\mu_k^2\right)}\right)^m. \tag{7b}$$

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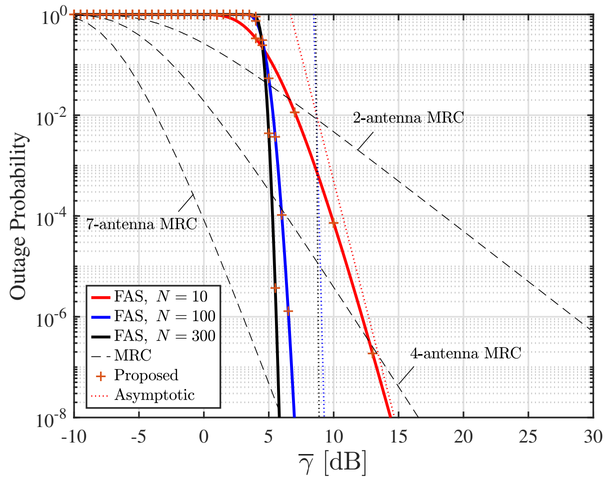

Fig. 2. OP vs. $\overline{\gamma}$, for different numbers of ports of FAS by assuming $W = 0.3$, $m = 1$, and $\gamma_{th} = 1$ dB. Markers denote the proposed approximation given in (6), whereas the solid and dotted lines represent the analytical and the asymptotic solutions given in (4) and (8), respectively.

Proof. See Appendix A. □

Remark 1. *Unlike previous contributions in the literature (e.g., [6, 8, 11]), where the OP was derived in integral form, the result in* (7) *is a simple and accurate approximation that does not need to solve any involved integrals regarding the joint distribution of correlated fading channels of FAS in order to reach the OP metric. Moreover,* (7) *is valid for the practical spatial correlation models proposed in [6].*

An asymptotic closed-form expression for the OP is derived to gain more insight into the impact of system parameters on the FAS performance. For that purpose, the asymptotic OP is expressed as $\mathrm{OP}^{\infty} \simeq \mathrm{G}_c \overline{\gamma}^{-\mathrm{G}_d}$ [13], where G_c and G_d are the array gain and the diversity order, respectively. The asymptotic OP is given in the following Proposition.

Proposition 2. *The asymptotic OP expression for the proposed FAS undergoing correlated Nakagami-m RVs is given by*

$$P_{\mathrm{out}}(\gamma_{th}) \simeq \frac{\left(\frac{\gamma_{th}}{\beta \overline{\gamma}}\right)^{\alpha}}{\alpha \Gamma(\alpha)}, \tag{8}$$

Proof. To asymptotically approximate (6), the relationship $\Upsilon(a, x) \simeq x^a / a$ as $x \to 0$ in (6), is employed. □

Remark 2. *From* (8)*, it is evident that the diversity order, $\mathrm{G}_d = Nm$, is directly affected by the number of ports and the severity of fading.*

IV. Numerical Results and Discussions

In this section, the effect of the system model parameters (e.g., antenna size, number of ports, and the severity of fading) on the OP performance in a FAS is addressed, as well as the goodness of the proposed approximation for the equivalent channel. Unless stated otherwise, for all figures, it is assumed that $\Omega_k = 1, \forall k$, and the spatial correlation model is formulated from (5). For the sake of comparison, the conventional MRC technique with uncorrelated antennas is included as a reference in the OP analysis.

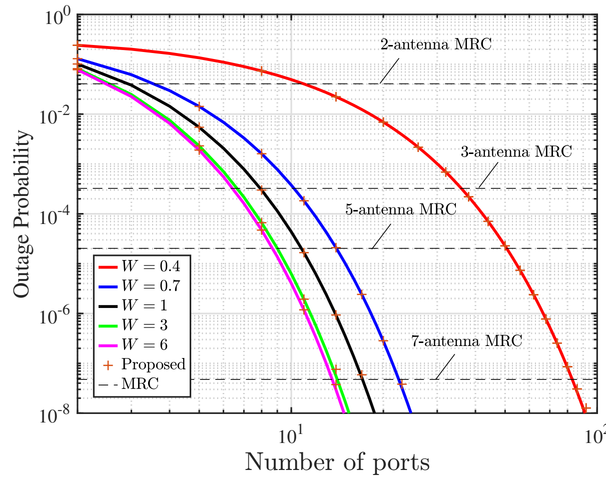

Fig. 3. OP vs. number of ports by varying W for $\overline{\gamma} = 5$ dB, $m = 1$, and $\gamma_{th} = 1$ dB. Solid lines denote the exact solution in (4).

In Fig 2, we illustrate the OP against the $\overline{\gamma}$ for different numbers of ports of FAS by setting $W = 0.3$, $m = 1$, and $\gamma_{th} = 1$ dB. In this case, the purpose is to demonstrate the accuracy of (6) to approximate the exact OP solution given in (4). Note that all figures show that the proposed approximations agree remarkably with the analytical ones for the whole average SNR range. It is worthwhile to mention that (4) is very demanding to calculate as the number of ports increases. Therefore, our approach with a simple mathematical fashion and negligible computational effort is highly attractive for further developments of FAS. On the other hand, based on the asymptotic plots, it is observed that the massive diversity of FAS contributes to the OP slope proportionally. This means that the OP decay is steeper (i.e., better OP performance is obtained) when the number of ports or the m parameter is increased (i.e., soft fading). Conversely, the OP is impaired as the number of ports or m decreases, and the decay is not so pronounced. These facts are in coherence with the results discussed in Remark 2. Note that the asymptotic OP quickly reaches the diversity order for $N = 10$. Contrariwise, for the cases with $N = 100, 300$, the asymptotic OP matches the true asymptotic behavior for extremely low operational OP values. In addition, a crossover in the OP is exhibited when $\overline{\gamma} < 5$ dB. For a better understanding of this behavior, Let us assume the case where the OP operates without fading, i.e., only the AWGN channel is considered. In this context, based on [15], the OP is equal to 1 for the SNR values below the outage threshold and identical to 0 otherwise. Now, by assuming fading channels (e.g., Nakagami-m), the OP < 1 for those SNR values below the outage threshold, whereas OP > 0 in the range of SNR values above such threshold. In other words, the slope of the OP does not decay as abruptly because of fading fluctuations (e.g., see the MRC curves). Based on this, for the Nakagami-m channel, as fading severity is soft (i.e., large m values), OP curves tend to reveal an OFF/ON behavior, similar to the AWGN case. From Remark 2, it is clear that the diversity order of the OP is proportional to fading severity and the number of ports, N. This explains the behavior of the curves below $\overline{\gamma} < 5$ dB, i.e., for large N

Table I. Comparison of computational efforts between the proposed approach and exact solution.
NMSE ranges from $-\infty$ (bad fit) to 1 (perfect fit).

Fig. #	Case	Average Elapsed Time (seconds): Proposed Asymptotic OP	Proposed OP	Exact OP	Time Reduction (%)	NMSE: Proposed OP
2	$N = 10$	0.0312	0.0781	24.554	99.68	0.99
	$N = 100$	0.1406	0.6562	141.90	99.53	0.99
	$N = 300$	0.4375	2.0937	391.26	99.46	0.99
	Memory in Use (megabytes)					
	Case	Proposed Asymptotic OP	Proposed OP	Exact OP	Memory Reduction (%)	
	$N = 10$	0.0095	0.0211	3.5810	99.41	
	$N = 100$	0.0121	0.0334	3.6123	99.07	
	$N = 300$	0.0238	0.0492	3.6726	98.66	

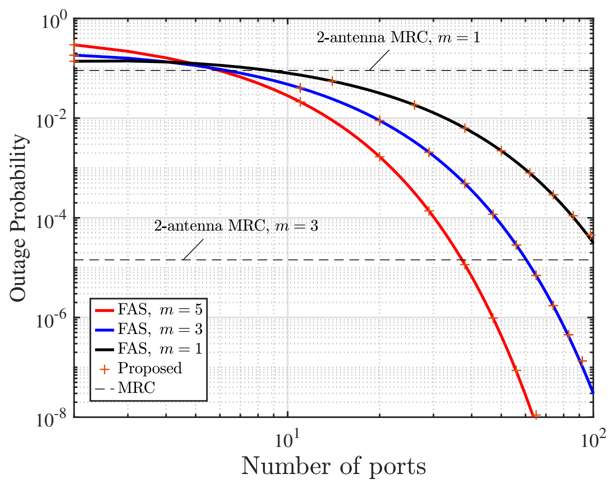

Fig. 4. OP vs. number of ports with $m = \{1, 3, 5\}$ for $W = 0.6$, $\overline{\gamma} = 3$ dB, and $\gamma_{th} = 1$ dB. Solid lines denote the exact solution in (4).

values, the OP plots behave like an AWGN channel, creating such a crossover. However, it is worth mentioning that, this behavior occurs in a non-operational range of SNR and OP of a standard communication system. Finally, the OP performance of FAS can meet or even exceed the MRC for high N values. In fact, this performance gap could be even more prominent in real-world rich-scattering environments with a large number of multipath clusters (e.g., an indoor area or a city street canyon). This is because the slope of the OP is governed by the Nm term, which means that large values of N or m (i.e., rich-scattering) lead to remarkable performance gains compared to classic MRC.

In Fig. 3, the OP is illustrated as a function of the number of ports of FAS for $\overline{\gamma} = 5$ dB, $m = 1$, and $\gamma_{th} = 1$ dB. In these curves, we explore the impact of varying the antenna size over the OP performance. It can be observed that large-size W coefficients (i.e., more space in the FAS) result in a better OP performance, as expected. Hence, the performance of FAS hugely relies on both the size W and the number of ports N of the fluid antenna at the FAS receiver.

Also, note that the OP performance is not restricted by W, i.e., no outage floor exists as N increases. On the other hand, FAS outperforms the MRC diversity scheme when the system is deployed with large antennas and a massive amount of ports.

In Fig. 4, the OP is illustrated as a function of the number

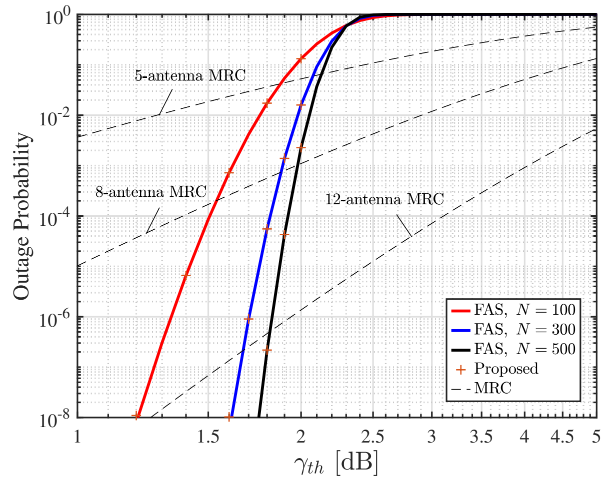

Fig. 5. OP vs. γ_{th} by varying N for $W = 2$, $\overline{\gamma} = 0$ dB, and $m = 1$. Solid lines denote the exact solution in (4).

of ports of FAS for $W = 0.6$, $\overline{\gamma} = 3$ dB, and $\gamma_{th} = 1$ dB. Herein, the achievable OP is investigated by varying the severity of fading, i.e., $m = \{1, 3, 5\}$. Three different scenarios for the OP behavior are observed regarding the configuration of parameters. Specifically, decreasing m (i.e., hard fading) harms the performance of the OP. Conversely, increasing m (i.e., mild fading condition) favors the OP. Furthermore, 2-antenna MRC for $m = 3$ has the lowest OP until FAS reaches $N = 37$ with $m = 5$. This MRC supremacy may be due to the power gain attributed to the active RF chains, while FAS has only one active RF chain.

Finally, Fig. 5 depicts the OP versus γ_{th} by varying N for $W = 2$, $\overline{\gamma} = 0$ dB, and $m = 1$. It can be observed that increasing γ_{th} leads to a significant loss in the OP. The best OP performance is achieved with massive ports in the FAS. As in the previous figure, the MRC scheme (i.e., 12-antenna configuration) outperforms the FAS behavior when dealing with higher γ_{th} values. However, this result can be reversed when the FAS combines a large antenna size together with many ports.

To quantify the fitting accuracy, we use the widely-accepted normalized mean-square-error (NMSE) test. Specifically, the NMSE measures the goodness-of-fit between the approximate and exact OPs, denoted by $\widehat{P}_{\text{out}}(\gamma_{th})$ and $P_{\text{out}}(\gamma_{th})$, respectively, i.e., NMSE $= 1 - ||P_{\text{out}}(\gamma_{th}) - \widehat{P}_{\text{out}}(\gamma_{th})||^2 / \left\| P_{\text{out}}(\gamma_{th}) - \mathbb{E}\left[P_{\text{out}}(\gamma_{th})\right] \right\|$, where $\mathbb{E}[\cdot]$ and $\|\cdot\|$ denote the expectation and the 2-norm operators, respectively. For informative purposes, Table I shows the computation time and memory in use of the proposed OP, asymptotic OP, and exact solutions of the illustrative examples considered in Figs. 2-3[4], when they are evaluated numerically. The reader can notice that our OP proposal is faster than the exact solutions and reduces the computational effort above 99.46% in all the examples regarding the exact ones. Concerning the asymptotic OP, it is noticeable that the elapsed time for the complete set of examples is negligible and less than one

[4]The computation of both the exact and the proposed OP expressions have been run in Windows 10 (64-bits) Pro Intel (R) Core (TM) i7-10510U - 2.9 GHz - 16 GB RAM.

second, influencing very slightly the number of ports in the elapsed time. Here, it can also be noted that memory in use reports an increase of more than 3 megabytes in memory consumption of the exact formulation compared with its counterpart in all cases examined, which makes our approach more attractive to run in any computing software. Likewise, Table I shows the NMSE between the approximate and exact OPs using the `goodnessOfFit` built-in function in MATLAB of the plots considered in Figs. 2-3. Results in Table I support the observation that our approach provides an outstanding fitting of the OP plots concerning the exact solutions, regardless of the value of N.

V. Conclusions

In this letter, we investigated the OP performance of a point-to-point FAS by assuming correlated Nakagami-m fading channels. Specifically, a novel asymptotic matching method is employed to approximate the CDF of FAS simply without incurring multi/single fold integrals. Then, with this result, a simple closed-form expression of the OP for the underlying system was obtained. Moreover, useful insights were provided regarding how fading channel conditions and the number of ports impact the OP performance of FAS. Finally, our results can be extended to FAS diversity schemes that still need to be explored in the literature.

Appendix A
Proof of Proposition 1

We first replace [16, Eq. (3)] into (4), so

$$P_{\text{out}}(\gamma_{th}) \approx \frac{2m^m}{\Gamma(m)\Omega_1^{2m}} \underbrace{\int_0^{\sqrt{\frac{\gamma_{th}}{\overline{\gamma}}}} r_1^{2m-1} \exp\left(-\frac{m r_1^2}{\Omega_1^2}\right)}_{I_1} \underbrace{\times \prod_{k=2}^{N} \left(\frac{\left(\frac{m\gamma_{th}}{\Omega_k^2\left(1-\mu_k^2\right)\overline{\gamma}}\right)^m \exp\left(-\frac{m\mu_k^2 r_1^2}{\Omega_1^2\left(1-\mu_k^2\right)}\right)}{m!} \right) dr_1}_{I_1}. \quad (9)$$

Here, with the aid of [17, Eq. (3.381.1)], applying a change of variables, and after some mathematical manipulations, I_1 can be evaluated in exact closed-fashion as

$$P_{\text{out}}(\gamma_{th}) \approx \frac{(m\gamma_{th})^m}{\Gamma(m)\Omega_1^{2m}\overline{\gamma}^m m!^{N-1}} \left(\frac{m\gamma_{th}\left(1+\sum_{k=2}^{N}\frac{\mu_k^2}{1-\mu_k^2}\right)}{\Omega_1^2\overline{\gamma}} \right)^{-m} \times \Upsilon\left(m, \frac{m\gamma_{th}\left(1+\sum_{k=2}^{N}\frac{\mu_k^2}{1-\mu_k^2}\right)}{\Omega_1^2\overline{\gamma}}\right) \prod_{k=2}^{N}\left(\frac{m\gamma_{th}}{\Omega_k^2(1-\mu_k^2)\overline{\gamma}}\right)^m. \quad (10)$$

Next, by applying both relationships $i)$ $\Upsilon(a,x) \simeq x^a/a$ as $x \to 0$, and $ii)$ $F_{g_{\text{FAS}}}(x) = P_{\text{out}}(x\overline{\gamma})$ into (10), the asymptotic behavior of the CDF of FAS in the form $F_{g_{\text{FAS}}}(x) \simeq a_0 x^{b_0}$, can be formulated as

$$F_{g_{\text{FAS}}}(x) \simeq \underbrace{\frac{m^{m-1}}{\Gamma(m)(\Omega_1)^{2m} m!^{N-1}} \prod_{k=2}^{N}\left(\frac{m}{\Omega_k^2(1-\mu_k^2)}\right)^m}_{a_0} x^{\overbrace{mN}^{b_0}}. \quad (11)$$

Then, (4) can be approximated with a Gamma distribution by utilizing the asymptotic matching method [12]. Therefore, the CDF and the asymptotic CDF of a Gamma approximation are respectively given by

$$\widetilde{F}_{g_{\text{FAS}}}(x) = \frac{\Upsilon\left(\alpha, \frac{x}{\beta}\right)}{\Gamma(\alpha)}, \widetilde{F}_{g_{\text{FAS}}}(x) \simeq \underbrace{\frac{1}{\beta^{\alpha}\alpha\Gamma(\alpha)}}_{\widetilde{a}_0} x^{\overbrace{\alpha}^{\widetilde{b}_0}}. \quad (12)$$

Then, by applying the asymptotic matching [12], i.e., $a_0 = \widetilde{a}_0$ and $b_0 = \widetilde{b}_0$, the shape parameters α and β can be expressed as (7). Finally, (6) is found with the help of (12) by setting $P_{\text{out}}(\gamma_{th}) \approx \widetilde{F}_{g_{\text{FAS}}}\left(\frac{\gamma_{th}}{\overline{\gamma}}\right)$. This completes the proof.

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