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Novel Expressions for the Outage Probability and Diversity Gains in Fluid Antenna System

José David Vega-Sánchez, *Member, IEEE*, Arianna Estefanía López-Ramírez, Luis Urquiza-Aguiar, *Member, IEEE*, and Diana Pamela Moya Osorio, *Senior Member, IEEE*

***Abstract*—The flexibility and reconfigurability at the radio frequency (RF) front-end offered by the fluid antenna system (FAS) make this technology promising for providing remarkable diversity gains in networks with small and constrained devices. Toward this direction, this letter compares the outage probability (OP) performance of non-diversity and diversity FAS receivers undergoing spatially correlated Nakagami-m fading channels. Although the system properties of FAS incur in complex analysis, we derive a simple yet accurate closed-form approximation by relying on a novel asymptotic matching method for the OP of a maximum-gain combining-FAS (MGC-FAS). The approximation is performed in two stages, the approximation of the cumulative density function (CDF) of each MGC-FAS branch, and then the approximation of the end-to-end CDF of the MGC-FAS scheme. With these results, closed-form expressions for the OP and the asymptotic OP are derived. Finally, numerical results validate our approximation of the MGC-FAS scheme and demonstrate its accuracy under different diversity FAS scenarios.**

***Index Terms*—Asymptotic matching, maximum-gain combining-FAS (MGC-FAS), nakagami-m fading, spatial correlation, outage probability.**

I. Introduction

Multiple-input multiple-output (MIMO) has been a fundamental part of the evolution of 5G to realize advancements in data rates and spectral efficiency. With MIMO, diversity gain is guaranteed as long as the antennas are separated by at least half wavelength. However, this may be challenging in very small devices of some Internet of Things (IoT) applications. Recently, a technology that uses liquid metals (e.g., gallium-indium eutectic) to design a software-controllable fluidic structure that, in its most basic implementation with only one radio frequency (RF) chain, allows a fluid radiator to switch among different positions in a small linear space, which has been referred to as a fluid antenna system (FAS) [1].

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(*Corresponding author: José David Vega-Sánchez*)

J. D. Vega-Sánchez is with the Faculty of Engineering and Applied Sciences (FICA), Telecommunications Engineering, Universidad de Las Américas (UDLA), Quito 170124, Ecuador (E-mail: jose.vega.sanchez@udla.edu.ec).

A. E. López-Ramírez, and L. Urquiza-Aguiar are with the Departamento de Electrónica, Telecomunicaciones y Redes de Información, Escuela Politécnica Nacional (EPN), Ladrón de Guevara E11-253, Quito, 170525, Ecuador. (e-mail: cecilia.paredes@epn.edu.ec; luis.urquiza@epn.edu.ec).

D. P. Moya Osorio is with the Communication Systems Division, Department of Electrical Engineering, Linköping University, Linköping 581 83, Sweden (e-mail: diana.moya.osorio@liu.se). She was also with the Centre for Wireless Communications, University of Oulu, Oulu 90014, Finland (e-mail: diana.moyaosorio@oulu.fi).

The performance of FAS has been investigated in a number of works. For instance, in [1], Wong et al. introduced the concept of a single-antenna FAS over correlated Rayleigh channels inspired by the advancement in mechanically flexible antennas. Afterward, in [2], Mukherjee et al. proposed a framework for the evaluation of the second-order statistic of the FAS by considering time-varying channels. In [3], Wong et al. revealed how the ergodic capacity scales with the system parameters of the FAS. In [4], Tlebaldiyeva et al. derived a single-integral form of the outage probability (OP) of a single-antenna FAS over correlated Nakagami-m fading channels. A concept of fluid antenna multiple access (FAMA) was proposed in [5], which takes advantage of the deep fades suffered by the interference to attain a good channel condition without complex signal processing. In [6], Skouroumounis et al. presented a framework based on stochastic geometry for evaluation the performance of large-scale FAS-aided cellular networks. In [7], New et al. investigated the limit of FAS performance and the diversity gain. In [8], Tlebaldiyeva et al. compared non-diversity and diversity FAS receivers over α-μ fading channels.Specifically, the diversity FAS scheme considers enabling multiple ports of a fluid antenna and using a combining technique with multi-port signals to enhance FAS performance further. Therein, a maximum-gain combining-FAS (MGC-FAS) scheme was investigated via Monte Carlo simulations due to the mathematical complexity for the underlying MGC-FAS.

Motivated by the potential of the FAS schemes to further enhance the capacity of future networks, with a great potential for IoT scenarios, we approximate the OP and asymptotic OP for the MGC-FAS scheme in a closed-form fashion, which is useful for evaluations of this scheme. For this purpose, we first approximate the cumulative density function (CDF) of each MGC-FAS branch, and then, the CDF of the MGC-FAS over correlated Nakagami-m fading is derived. In both stages, the fitting parameters are estimated by employing the asymptotic matching method, proposed in [9] that render a simple yet accurate approximation. To the best of the author's current knowledge, no prior work has provided a closed-form expression for the OP of the MGC-FAS scheme in the literature.

II. System and Channel Models

Consider a point-to-point FAS where the transmitter is equipped with a traditional antenna and the receiver with a fluid one with/without a diversity scheme, as described below.

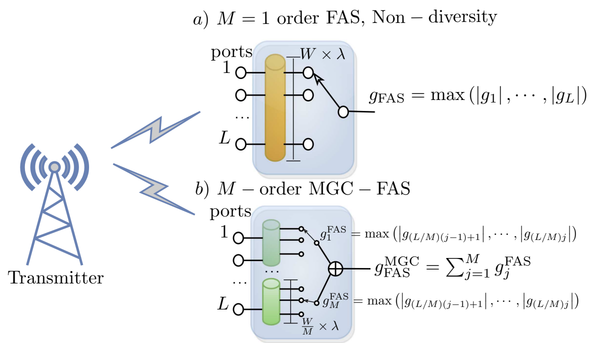

Fig. 1. System model for FAS-enabled communication with a) MGC-FAS scheme and b) non-diversity FAS configuration.

A. Non-Diversity FAS Receiver

Here, the FAS-receiver is built of a fluid antenna that can move freely along L-ports equally distributed along a linear dimension of length $W\lambda$, with W being the antenna size and λ the wavelength of the carrier frequency, as illustrated in Fig. 1a. We assume that the FAS can always switch to the best port for reaching the maximum received signal-to-noise ratio (SNR). Thus, the channel gain for the FAS is given by

$$g_{\text{FAS}} = \max\left(|g_1|, \cdots, |g_L|\right), \tag{1}$$

where $g_i = |h_i|^2$ for $i \in \{1, \cdots, L\}$ denotes the channel gain of each port in the FAS, with h_i being modeled as correlated Nakagami-m fading because the antennas are located very close to each other in the linear space. The received SNR for non-diversity FAS receiver can be formulated as

$$\gamma = \frac{P\,|g_{\text{FAS}}|}{N_0} = \overline{\gamma}\,|g_{\text{FAS}}|, \tag{2}$$

where $\overline{\gamma} = \frac{P}{N_0}$ is the average transmit SNR, with P being the transmit power and N_0 the additive white Gaussian noise (AWGN) power.

B. Diversity FAS Receiver

The entire FAS with $W \times \lambda$ is split into M sub-FAS branches with a size of $W \times \lambda/M$, so the MGC-FAS comprises L/M ports per FAS branch, as shown in Fig. 1b. In the MGC-FAS scheme, the end-to-end channel is the sum of the strongest channels of each FAS tube as

$$g_{\text{FAS}}^{\text{MGC}} = \sum_{j=1}^{M} g_j^{\text{FAS}}, \tag{3}$$

where $g_j^{\text{FAS}} = \max\left(\left|g_{(L/M)(j-1)+1}\right|, \cdots, \left|g_{(L/M)j}\right|\right)$, and $g_k = |h_k|^2$ for $k \in \{(L/M)(j-1)+1, \cdots, (L/M)j\}$, with h_k subject to correlated Nakagami-m fading. The received SNR for MGC-FAS receiver can be expressed as

$$\gamma^{\text{MGC}} = \frac{P\left|g_{\text{FAS}}^{\text{MGC}}\right|}{N_0} = \overline{\gamma}\left|g_{\text{FAS}}^{\text{MGC}}\right|. \tag{4}$$

In the following sections an approximate statistical model for $g_{\text{FAS}}^{\text{MGC}}$ will be obtained, then the OP distribution can be obtained.

III. Performance Analysis

The OP is considered to assess the performance of the FAS, which is defined as the probability that the received SNR is less than a threshold rate γ_{th}.

A. Exact OP Distributions

Diversity case: From [4, Eq. (10)] and applying the relationship $F_{g_j^{\text{FAS}}}(x_j) = P_{\text{out}}(x_j\overline{\gamma})$ for $j \in \{1, \cdots, M\}$, the CDF of the end-to-end channel gain in each sub-FAS branch of (3) is given by

$$F_{g_j^{\text{FAS}}}(x_j) = \frac{2^m m}{\Gamma(m)\Omega_1^{2m}} \int_0^{\sqrt{x_j}} r_1^{2m} \exp\left(-\frac{m r_1^2}{\Omega_1^2}\right) \times \prod_{k=2}^{L/M}\left(1 - Q_m\left(\sqrt{\frac{2m\mu_k^2 r_1^2}{\Omega_1^2\left(1-\mu_k^2\right)}}, \sqrt{\frac{2m x_j}{\Omega_k^2\left(1-\mu_k^2\right)}}\right)\right) dr_1, \tag{5}$$

where $\Gamma(\cdot)$ denotes the Gamma function, $Q_m(\cdot,\cdot)$ is the m-order Marcum Q-function, m is the fading parameter, and Ω_k^2 indicates the average channel power. Motivated by [10], we assume that the spatial correlation coefficient, denoted by, μ_i, is given by

$$\mu^2 = \left|\frac{2}{L(L-1)} \sum_{i=1}^{L-1} (L-i) J_0\left(\frac{2\pi i W}{L-1}\right)\right|, \textit{ for } \mu = \mu_i \ \forall i, \tag{6}$$

where all the ports don't have a reference port or any port is a reference to any other port. Also, $J_0(\cdot)$ is the zero-order Bessel function of the first kind. Then, the probability density function (PDF) of (5), i.e., $f_{g_j^{\text{FAS}}}(x_j)$ can be obtained by computing the respective derivative. To obtain the CDF of (3), we use Brennan's approach[1], which argued that when all the sub-FAS branches involved in the CDF sum of the MGC-FAS scheme are non-negative random variables (RVs) (like our case of fading envelopes), the traditional convolution to computed such a CDF can be reformulated within its limits of integration employing a geometric approach as an M-fold integral in terms of the joint PDF of the correlated branches. Specifically, the distribution of the sum of M RVs is the integral of the joint correlated density function over the M-dimensional volume bounded by the hyper-plane $x_1 + x_2 + \cdots + x_M = x$ and the coordinate hyper-planes. Based on this, the end-to-end CDF of the MGC-FAS in (3) is expressed as

$$F_{g_{\text{FAS}}^{\text{MGC}}}(x) = \int_0^x \int_0^{x-x_M} \cdots \int_0^{x-\sum_{i=3}^{M} x_i} \int_0^{x-\sum_{i=2}^{M} x_i} \times f_{g_j^{\text{FAS}},\ldots,g_M^{\text{FAS}}}(x_1,\ldots,x_M)\,dx_1 dx_2 \ldots dx_{M-1} dx_M, \tag{7}$$

where $f_{g_j^{\text{FAS}},\ldots,g_M^{\text{FAS}}}(x_1,\ldots,x_M)$ is the joint PDF of the correlated branches. Finally, the OP for the MGC-FAS is computed as $P_{\text{out}}^{\text{MGC}}(\gamma_{th}) = F_{g_{\text{FAS}}^{\text{MGC}}}\left(\frac{\gamma_{th}}{\overline{\gamma}}\right)$.

Non-diversity case: From (5), the OP for non-diversity FAS over correlated Nakagami-m RVs is given as $P_{\text{out}}(\gamma_{th}) = F_{g_j^{\text{FAS}}}\left(\frac{\gamma_{th}}{\overline{\gamma}}\right)$ by setting $M=1$.

B. Proposed OP Approximation

It is noteworthy that the multi-fold integral in (7) is quite intricate, thus the derivation of a closed-form solution appears to be unfeasible. To overcome such limitation, an accurate approximation is proposed for the $P_{\text{out}}^{\text{MGC}}(\gamma_{th})$, which is

[1] Interested readers can refer to [11, Secs. IV-V] for detailed information on Brenann's approach.

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obtained via the asymptotic matching method introduced in [9], as stated in the following proposition.

Proposition 1. *The OP expression of FAS undergoing correlated Nakagami-m RVs can be approximated by*

$$P_{\text{out}}^{\text{MGC}} \approx \frac{\Upsilon(\alpha_{\text{MGC}}, \frac{\gamma_{th}}{\beta_{\text{MGC}}\overline{\gamma}})}{\Gamma(\alpha_{\text{MGC}})}, \tag{8}$$

where is $\Upsilon(\cdot,\cdot)$, is the lower incomplete gamma function [12, Eq. (6.5.2)] and

$$\alpha_{\text{MGC}} = \frac{L}{M}\sum_{j=1}^{M} m_j, \quad \beta_{\text{MGC}} = \left(\frac{1}{\prod_{j=1}^{M}\frac{1}{(\beta_j^{\text{FAS}})^{\alpha_j^{\text{FAS}}}}}\right)^{1/\alpha_{\text{MGC}}}, \tag{9a}$$

$$\alpha_j^{\text{FAS}} = \tfrac{L}{M} m_j, \quad \beta_j^{\text{FAS}} = \left(\frac{1}{\Gamma(\alpha_j^{\text{FAS}}) a_{0,j} \alpha_j^{\text{FAS}}}\right)^{1/\alpha_j^{\text{FAS}}}, \tag{9b}$$

$$a_{0,j} = \frac{m_j^{m_j-1}}{\Gamma(m_j)\Omega_{1,j}^{2m_j}(m_j!)^{\frac{L}{M}-1}} \prod_{k=2}^{\frac{L}{M}} \left(\frac{m_j}{\Omega_{k,j}^2\left(1-\mu_{k,j}^2\right)}\right)^{m_j}. \tag{9c}$$

Proof. See Appendix A. □

Remark 1. *Notice that the OP expression is a novel and simple approximation that does not need to solve any involved integrals regarding the joint distribution of correlated fading channels of the MGC-FAS scheme.*

In order to attain more insights into the influence of system parameters for the MGC-FAS performance, an asymptotic closed-form expression for the OP is derived. To this end, the asymptotic OP is developed in the form $\text{OP}^{\infty} \simeq \text{G}_c \overline{\gamma}^{-\text{G}_d}$ [13], where G_c and G_d is the array gain and the diversity order, respectively. The asymptotic OP is stated in the following Proposition.

Proposition 2. *The asymptotic OP expression for the proposed MGC-FAS over correlated Nakagami-m RVs is given by*

$$P_{\text{out}}^{\text{MGC}}(\gamma_{th}) \simeq \frac{(\frac{\gamma_{th}}{\beta_{\text{MGC}}\overline{\gamma}})^{\alpha_{\text{MGC}}}}{\alpha_{\text{MGC}}\Gamma(\alpha_{\text{MGC}})}, \tag{10}$$

Proof. By using $\Upsilon(a,x) \simeq x^a/a$ as $x \to 0$ into (8), (10) is obtained straightforwardly. □

Remark 2. *From* (10) *and* (9a)*, it is clear that the diversity order reduces to $\text{G}_d = Lm$ when all the sub-FAS tubes experience the same fading, i.e., $m = m_i$, $\forall i$. Moreover, G_d is directly influenced by the number of ports and the fading severity.*

IV. Numerical Results and Discussions

In this section, the impact of the system model parameters (e.g., the number of M sub-FAS branches, the size of the antenna W, and the severity of fading) on the OP performance is investigated, as well as the accuracy of the proposed approximations through illustrative examples. Unless stated otherwise, $\Omega_k = 1, \forall k$ is considered for all plots, and the spatial correlation model is computed with the help of (6).

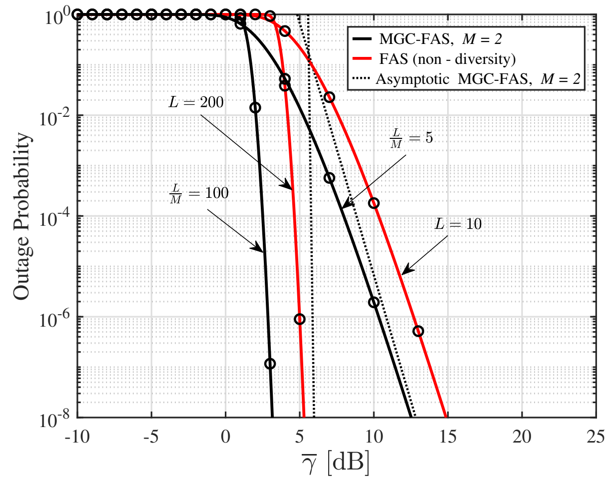

Fig. 2. OP vs. $\overline{\gamma}$, for different numbers of ports by assuming $W = 2$, $m = 1$, and $\gamma_{th} = 5$ dB. Markers denote the proposed approximation given in (8), whereas the solid and dotted lines represent the analytical and the asymptotic solutions computed via (7) and (10), respectively.

For the sake of comparison, $i)$ the traditional maximal ratio combining (MRC) technique with uncorrelated antennas and $ii)$ the non-diversity FAS receiver, are included as a baseline in the OP analysis.

In Fig 2, we show the OP versus the $\overline{\gamma}$ for $W = 2$, $m = 1$, $\gamma_{th} = 5$ dB and by varying the number of ports $L = \{10, 200\}$ in the non-diversity FAS receiver. Hence, when setting $M = 2$-order MGC-FAS, it means that there are two sub-FAS tubes with $\frac{L}{M} = \{5, 100\}$ ports each. In this figure, the accuracy of the proposed approach in (8) to approximate the exact solution computed via (7) is evaluated. Note that all approximate curves are tight to the analytical solutions for the entire average SNR range. It is worth noting that as M (i.e., the FAS tubes) increases, the exact formulation in (7) becomes computationally hard to compute, prone to convergence, or even unworkable. Hence, our simple and accurate approximation, with negligible computational cost, proves useful for the performance analysis for diversity FAS schemes. Considering the asymptotic curves, note that the OP decline is steeper (i.e., good OP performance) as the number of ports in the sub-FAS tubes or the severity fading m increases. Contrariwise, the OP is affected when the number of sub-FAS ports or the m parameter decreases, so the OP slope is is less pronounced. These results are in coherence with the insights examined in Remark 2. On the other hand, it is observed that the asymptotic OP of the MGC-FAS quickly matches the diversity order of the exact solution for $\frac{L}{M} = 5$. Conversely, for the scenario with $\frac{L}{M} = 5$, the asymptotic OP fits the correct asymptotic behavior for relatively lower operational OP values.

Henceforth, the approximation curves are represented in all plots with solid lines for visibility purposes. In Fig. 3, the OP is depicted as a function of the number of ports of the FAS scheme. For instance, the fixed value $L = 100$ on the x-axis for a non-diversity FAS scheme corresponds to $\frac{L}{M} = 50$ in the 2-order MGC-FAS technique. The remaining parameters are set to: $\overline{\gamma} = 1$ dB, $m = 1$, and $\gamma_{th} = 2$ dB. In these results, the influence of changing the antenna size W

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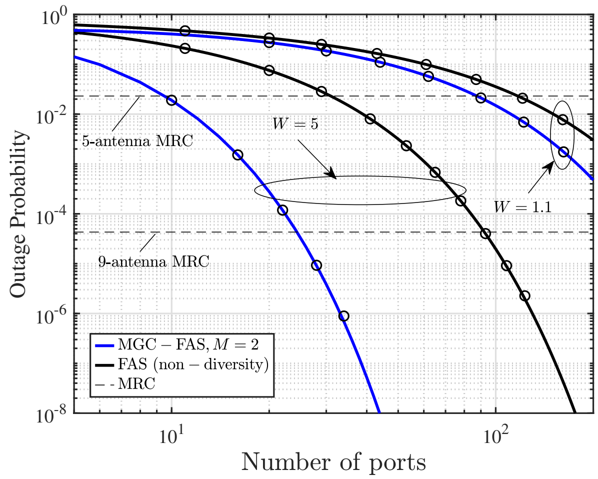

Fig. 3. OP vs. number of ports by varying W for $\overline{\gamma} = 1$ dB, $m = 1$, and $\gamma_{th} = 2$ dB. Markers denote Monte Carlo simulations, whereas the solid lines denote the proposed approximations given in (8).

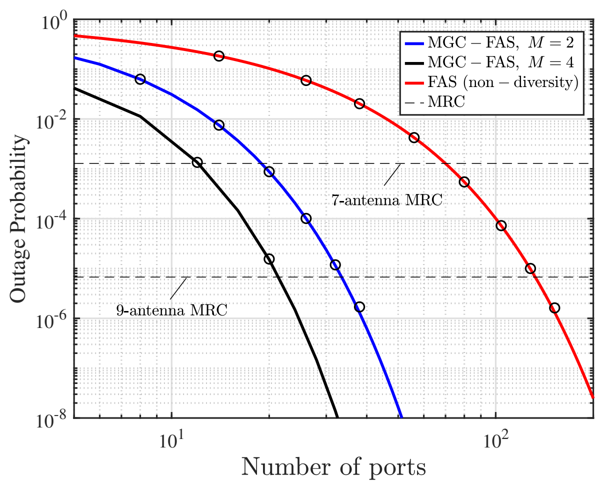

Fig. 4. OP vs. number of ports with $M = \{2, 4\}$-order MGC-FAS for $W = 3$, $m = 1$, $\overline{\gamma} = 1$ dB, and $\gamma_{th} = 2$ dB. Markers denote Monte Carlo simulations, whereas the solid lines denote the proposed approximations given in (8).

on the OP behavior for MGC-FAS receivers is investigated. Note that large W values (i.e., more space in the MGC-FAS) favor the OP performance compared to the non-diversity FAS scheme. In particular, obtaining a higher performance gain from MGC-FAS over non-diversity FAS highly depends on the size coefficient W for a fixed number of ports. Moreover, non-diversity/diversity FAS schemes beat the MRC method. For instance, the OP for the 9-antenna MRC is exceeded when the FAS is deployed by assuming $W = 5$, $L = 91$, and $\frac{L}{M} = 24$ ports for non-diversity FAS and MGC-FAS schemes, respectively.

In Fig. 4, the OP is displayed as a function of the number of ports, as explained in Fig. 3. Herein, the achievable OP is examined by comparing $M = \{2, 4\}$-order MGC-FAS with the non-diversity FAS receiver for $W = 3$, $m = 1$, $\overline{\gamma} = 1$, and $\gamma_{th} = 2$ dB. All plots show that increasing the sub-FAS tubes (i.e., M) greatly benefits the performance of the OP compared to the non-diversity FAS receiver. In fact, this gain

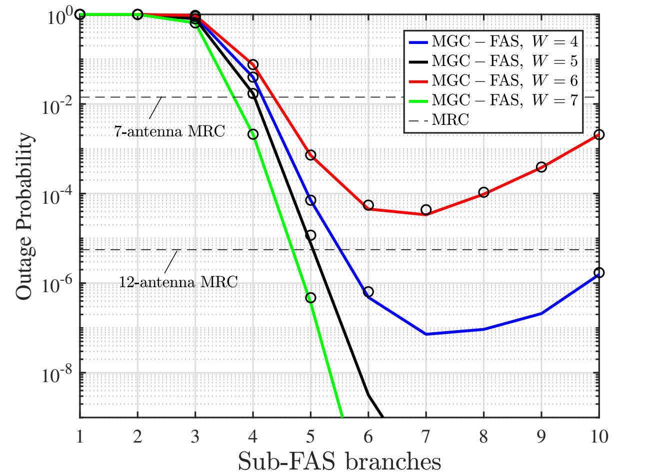

Fig. 5. OP vs. the number of sub-FAS branches M by varying the antenna size W for $L = 200$, $\overline{\gamma} = 0$ dB, and $m = 1$. Markers denote Monte Carlo simulations, whereas the solid lines denote the proposed approximations given in (8).

gap between M-order MGC-FAS and the FAS could be further boosted by increasing the size of the antenna W, as explained in Fig. 3. Furthermore, 9-antenna MRC is surpassed when the FAS is assumed with $\frac{L}{M} = 20$, $\frac{L}{M} = 34$ and $L = 132$ ports for 4-order MGC-FAS, 2-order MGC-FAS, and non-diversity FAS schemes, respectively. This fact confirms the importance of using diversity in the FAS receiver.

Fig. 5 illustrates the OP versus the number of sub-FAS branches[2], i.e., M by varying the antenna size W for $L = 200$, $\overline{\gamma} = 0$ dB, and $m = 1$. Here, our primary goal is to provide a general thought on determining the more suitable number of branches in the FAS diversity scheme to further improve OP's performance. Overall, it can be noticed that increasing the number of sub-FAS branches M leads to a remarkable improvement in the OP's performance. However, this OP's behavior is up to a break-point where increasing M instead of helping harms the OP. This is because large numbers of sub-FAS branches $(M \uparrow)$ distributed into a small antenna size $(W \downarrow)$ lead to highly correlated ports, which causes the gain of the combining selection technique to be negligible. In addition, it can be observed that the appearance of such a break-point occurs at very low OP values when dealing with large W values (e.g., $W = 6, 7$). Finally, MRC configurations are beaten more quickly using higher-order MGC-FAS schemes as long as the break-point is not exceeded.

V. Conclusions

In this letter, we examined the OP performance of a point-to-point FAS by assuming non-diversity and diversity FAS receivers undergoing correlated Nakagami-m fading channels. Specifically, a novel asymptotic matching method is employed to approximate the CDF of the MGC-FAS receiver in an analytically tractable way without incurring multi/single-fold integrals. With this by-product, a simple closed-form expression of the OP for the MGC-FAS scheme was derived. Fur-

[2]It is worth mentioning that for both the Monte Carlo simulations and the analytical approximations, $\lfloor L/M \rfloor$ is assumed, where $\lfloor \cdot \rfloor$ is the floor function.

thermore, useful insights were provided concerning how the antenna size W influences the OP performance of the MGC-FAS. Specifically, the MGC-FAS scheme provided remarkable gains in terms of the OP over the non-diversity FAS when the W values are large enough.

Appendix A
Proof of Proposition 1

In the first stage, a suitable approximation for the CDF of each sub-FAS branch given in (5) is obtained by using a Gamma distribution. Toward that [14, Eq. (3)] is replaced into (5), which is re-expressed as

$$F_{g_j^{\mathrm{FAS}}}(x_j) \approx \frac{2{m_j}^{m_j}}{\Gamma(m_j)\Omega_{1,j}^{2m_j}} \underbrace{\int_0^{\sqrt{x_j}} r_1^{2m_j-1}\exp\left(-\tfrac{m_j r_1^2}{\Omega_{1,j}^2}\right)}_{I_1} \underbrace{\times \prod_{k=2}^{\frac{L}{M}} \left(\frac{\left(\frac{m_j x_j}{\Omega_{k,j}^2\left(1-\mu_{k,j}^2\right)}\right)^{m_j} \exp\left(-\frac{m_j \mu_{k,j}^2 r_1^2}{\Omega_{1,j}^2\left(1-\mu_{k,j}^2\right)}\right)}{m_j!} \right) dr_1}_{I_1}. \tag{11}$$

With the aid of [15, Eq. (3.381.1)], I_1 can be evaluated in closed-fashion in terms of the incomplete Gamma function, i.e., $\Upsilon(\cdot,\cdot)$. Then, by applying, $\Upsilon(a,x) \simeq x^a/a$ as $x \to 0$, the asymptotic behavior of the CDF of each sub-FAS branch in the form $F_{g_j^{\mathrm{FAS}}}(x_j) \simeq a_0 x_j^{b_0}$, can be formulated as

$$F_{g_j^{\mathrm{FAS}}}(x_j) \simeq \underbrace{\frac{m_j^{m_j-1} m_j!^{1-\frac{L}{M}}}{\Gamma(m_j)(\Omega_{1,j})^{2m_j}} \prod_{k=2}^{\frac{L}{M}} \left(\frac{m_j}{\Omega_{k,j}^2(1-\mu_{k,j}^2)}\right)^{m_j}}_{a_0} x_j^{\overbrace{m_j \frac{L}{M}}^{b_0}}. \tag{12}$$

To find the shape parameters of the Gamma distribution to approximate (5), the asymptotic Gamma CDF[3] of each sub-FAS branch is used to obtain the following expression

$$\widetilde{F}_{g_j^{\mathrm{FAS}}}(x_j) \simeq \underbrace{\frac{1}{\beta_j^{\mathrm{FAS}\,\alpha_j^{\mathrm{FAS}}} \alpha_j^{\mathrm{FAS}} \Gamma(\alpha_j^{\mathrm{FAS}})}}_{\widetilde{a}_0} x_j^{\overbrace{\alpha_j^{\mathrm{FAS}}}^{\widetilde{b}_0}}. \tag{13}$$

Then, by applying the asymptotic matching [16], i.e., $a_0 = \widetilde{a}_0$ and $b_0 = \widetilde{b}_0$, the shape parameters α_j^{FAS} and β_j^{FAS} of the Gamma distribution to approximate (5) can be expressed as (9b). In the second stage, we approximate the CDF of the MGC-FAS in (7) by using again the Gamma distribution via the asymptotic matching technique. For this purpose, the approximate PDF and CDF of the MGC-FAS can be expressed as

$$\widetilde{f}_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}(x) = \underbrace{\frac{x^{\overbrace{\alpha_{\mathrm{MGC}}-1}^{\widetilde{d}_0}}}{\Gamma\left(\alpha_{\mathrm{MGC}}\right)\beta_{\mathrm{MGC}}{}^{\alpha_{\mathrm{MGC}}}}}_{\widetilde{c}_0} \exp\left(-\frac{x}{\beta_{\mathrm{MGC}}}\right) \tag{14a}$$

$$\widetilde{F}_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}(x) = \frac{\Upsilon(\alpha_{\mathrm{MGC}}, \frac{x}{\beta_{\mathrm{MGC}}})}{\Gamma(\alpha_{\mathrm{MGC}})}, \tag{14b}$$

where $\widetilde{c}_0$ and $\widetilde{d}_0$ are the linear and the angular coefficients that capture the asymptotic behavior of the approximate distribution, i.e., $\widetilde{f}_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}(x)$. Now, we are interested in the asymptotic behavior of the PDF of the sum $f_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}(x)$ given in (3). Hence, by appropriately substituting the shape parameters of the summands, i.e., α_j^{FAS} and β_j^{FAS} into [16, Eq. (4)] and after some manipulations, the linear and angular coefficients that govern the asymptote of the sum, $f_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}(x) \simeq c_0 x^{d_0}$, are given by

$$c_0 = \frac{\prod_{j=1}^{M}\left[\frac{1}{\Gamma\left(\alpha_j^{\mathrm{FAS}}\right)\beta_j^{\mathrm{FAS}\,\alpha_j^{\mathrm{FAS}}}}\right]}{\Gamma\left(\sum_{j=1}^{M}\alpha_j^{\mathrm{FAS}}\right)}, \quad d_0 = -1 + \sum_{j=1}^{M}\alpha_j^{\mathrm{FAS}}. \tag{15}$$

Next, by matching $c_0 = \widetilde{c}_0$ and $d_0 = \widetilde{d}_0$, the fitting parameters of (14) are found straightforwardly. Finally, (8) is obtained with the help of (14b) by setting $P_{\mathrm{out}}^{\mathrm{MGC}}(\gamma_{th}) \approx \widetilde{F}_{g_{\mathrm{FAS}}^{\mathrm{MGC}}}\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)$. This completes the proof.

[3]To asymptotically approximate $F_{g_j^{\mathrm{FAS}}}(x_j) \approx \frac{\Upsilon\left(\alpha_j^{\mathrm{FAS}}, \frac{x_j}{\beta_j^{\mathrm{FAS}}}\right)}{\Gamma(\alpha_j^{\mathrm{FAS}})}$, the relationship $\Upsilon(a,x) \simeq x^a/a$ as $x \to 0$, is employed.

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