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Model predictive controlled subsurface drainage and irrigation for peatland groundwater management ⋆

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Abstract: A dynamic simulation model for the water balance in a field with controlled subsurface drainage and irrigation is derived. The model is then applied to groundwater table height control, using the model predictive control (MPC) paradigm. The nonlinear constrained MPC problem is solved and results illustrated in simulations. The role of automation of controlled subsurface drainage in cultivated peatlands is discussed.

Keywords: modeling and control of agriculture; man-machine systems in agriculture; decision support systems; climate change impact and adaptation measures; optimal control and operation of water resources systems

1. INTRODUCTION

There is a critical necessity to refine existing cultivation practices and develop new digital solutions in order to fuse practical cultivation needs, carbon sequestration, mitigation of greenhouse gas (GHG) emissions and water loading, and availability of water resources. Increasing the height of groundwater table is an effective way to regulate GHG emissions from cultivated peatlands (Evans et al. (2021)). Simultaneously, controlled water management allows agriculture to adapt to changing weather conditions and enables efficient use of available water. The practical needs in cultivation stem from the increased size of farms, expansion in the amount of responsibilities on farmers, and needs for a holistic, timely and anticipating control and maintenance of all the resources.

In Finland, only 15 % of the land can be cultivated with no drainage. Most of the fields in Finland are subsurface drained (Salo (2019)). During past decades, the interest in controlled subsurface draining has increased, as well as considerations for the application of draining pipes for irrigation. With the emerging availability of affordable IoT sensors and remotely controlled valve systems, with local energy supply and robust data transfer abilities, the control of the subsurface drainage systems is in transition from passive manual operations to active semi or fully automated control.

Modern approaches to automatic control are model-based. Modeling of water balance in controlled subsurface drainage is complicated, however, due to the heterogeneousity of various field sectors, and the numerous practical requirements posed on the models and tuning. Control is further complicated by the immature infrastructure and conflicts in subgoals. In this paper, the control dynamics of water balance in cultivated peatlands are of importance, leading to that detailed approaches such DRAINMOD (Skaggs et al. (2012)) will not be feasible due to computational load and needs in calibrating the model. Recent papers with similar goals include van der Craats (2021), where a list of papers and references on the topic is also provided. The model presented in van der Craats (2021) is still containing a relatively large number of parameters, their optimization approach is relying on Monte Carlo simulations and a set of heuristic rules. Model Predictive Control (MPC) in water management has been considered e.g. by Ayaz et al. (2020), who focused on modelling and control of canal pools. The urban runoff was examined in Oberascher et al. (2021); Lund et al. (2018) provide a relatively recent oveview on MPC of urban drainage systems. This paper proposes a novel semi-physical model for cultivated lands, based on a 1-D mass balance, aiming at realistic tunability, dynamics, and integration of well and drainage pipe systems for drainage and irrigation.

In the reminder of this paper, a straightforward modeling and nonlinear constrained MPC approach for subsurface controlled drainage and irrigation of cultivated peatlands is proposed. In Section 2, a dynamic water balance is derived, to be used for predictions in MPC, discussed in Section 3. Numerical experiments in Section 4 illustrate the approach and the paper ends with conclusions.

2. DYNAMIC WATER BALANCE

The main purpose of the derived model is to enable predicting the ground water level dynamics for a few weeks ahead, given the field current state, weather forecasts, drainage and irrigation control actions, as well as crop growth stage and other major parameters impacting the water balance. The model should be simple enough for on-line computations, especially aiming at MPC purposes, and allow feasible parameter tuning and state estimation

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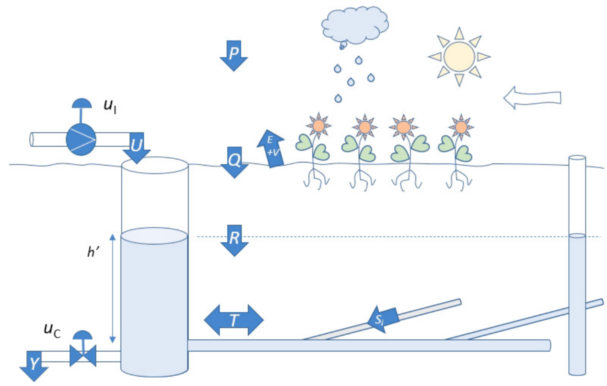

Fig. 1. 1-D water mass balance for cultivated peatland with controlled subsurface drainage and irrigation.

(data assimilation) using commonly available field parameters and on-line data.

The developed dynamic model consists of four parts: i) characterization of the precipitation that enters the unsaturated soil; ii) dynamics of the flux of water through the unsaturated zone of soil; iii) dynamics of the groundwater storage and table height and iv) dynamics of the subterrain drainage well with input and output control actuators. Fig. 1 illustrates the concept and notations. The 1D-model is derived based on basic water mass balances, exploiting the Hooghoudt drainage equation for subsurface drainage flows.

2.1 Precipitation

A mass balance for the water entering the ground is constructed as

$$\dot{m}_P(t) = \dot{m}_Q(t) + \dot{m}_E(t) + \dot{m}_V(t)$$

In the notation, $\dot{m}$ denotes mass flow [kg/s]. Subscript P denotes the precipitation from rain (or surface irrigation, or alike); E the evapotranspiration, a joint impact of evaporation from the ground surface and consumption by the plants; V the combined impact of other factors: surface runoff, etc. The $\dot{m}_Q$ is the water flux entering the unsaturated soil zone.

Denote by X [m/s] a one-dimensional volumetric flow (commonly expressed in meteorology in mm/day). Since $\dot{m} = \rho A X$ [kg/s] and spatially uniform precipitation is assumed, where A [m^2] is the surface area and ρ [kg/m^3] the density of water, the mass balance can be expressed in terms of one-dimensional volumetric flows:

$$Q(t) = P(t) - E(t) - V(t) \tag{1}$$

Data on precipitation P [m/s] is available from weather statistics and forecasts. Evapotranspiration E [m/s] can be estimated based on crop growth stage and solar irradiation (possibly using additional knowledge on air humidity, wind, leaf indexes, etc.), available from weather and farming statistics and forecasts. V [m/s] is normally zero, but can be estimated to be nonzero, e.g. during heavy rain, flooding, etc. Q [m/s] is the surface precipitation that will eventually enter the ground water table. In what follows, it is assumed that Q can be estimated based on prior knowledge and the dynamics can be neglegted.

2.2 Unsaturated zone

It is assumed that the water enters the unsaturated zone only from the surface. The mass balance for the zone

$$\frac{dm}{dt}(t) = \dot{m}_{in}(t) - \dot{m}_{out}(t)$$

can then be constructed as

$$\epsilon \rho A \frac{dR}{dt}(t) = \frac{K}{d - h(t)}\left(\rho A Q(t) - \epsilon \rho A R(t)\right)$$

leading to a first-order dynamic equation with a time-varying time-constant

$$\frac{dR}{dt}(t) = \frac{K}{d - h(t)}\left(\frac{1}{\epsilon}Q(t) - R(t)\right) \tag{2}$$

describing the dynamics of the ground water recharge R [m/s]. The coefficient ϵ [-] is the volumetric portion of the water impacted by subsurface drainage to the total volume, the reminder consists, e.g., of stones and water that is not removed (at least immediately) by the pressure differences involved in subsurface drainage. ϵ can be estimated from soil water retention curves. ρ [kg/m^3] and A [m^2] denote the density of water and ground surface of the considered field sector, respectively. The time constant for the dynamics is assumed to depend on the hydrological conductivity of the soil K [m/s], and inversely on the distance for the water to traverse through the unsaturated soil $d-h$, where d [m] is the distance from the subsurface drains to the surface and h [m] the height of the ground water table from the same. K can be estimated based on the soil type, d is a field parameter.

2.3 Ground water table

The ground water zone is located below the unsaturated zone. The ground water storage mass balance reads as

$$\epsilon \rho A \frac{dh}{dt}(t) = \epsilon \rho A R(t) - \rho S(t)$$

and solving for change in the water table height h gives

$$\frac{dh}{dt}(t) = R(t) - \frac{1}{\epsilon A} S(t) \tag{3}$$

where S [m^3/s] is the field drainage, or irrigation from drainage well if negative. This flow takes place via the suction and collection pipes between field sectors and well, the dynamics (transients and delays) of which are assumed to be so fast that they can be omitted. The flow is estimated using the Hooghoudt's drainage equation (Waller and Yitayew (2016)), often expressed as

$$SL^2 = 4K_a\tilde{h}^2 + 8K_b\tilde{h}d_e$$

where K_a and K_b [m/s] are the hydraulic conductivities of the soil above and below the drain level (for simplicity $K = K_a = K_b$ can be assumed) and d_e [m] is the equivalent depth to impermeable layer. L [m] is the spacing between drains. Hooghoudt's drainage equation has been developed for steady state drainage, but is assumed valid here due to slow changes in water table height. In uncontrolled drainage, the head difference $\tilde{h}$ is given by $\frac{4}{\pi}(h-d)$. The $\frac{4}{\pi}$ coefficient comes from the fact that the Hooghoudt equation uses the height at the midpoint between drains, which can be approximated from the mean water level h

assuming a conic shape for the water table. In controlled drainage,

$$\tilde{h} = \frac{4}{\pi}(h - h') \tag{4}$$

can be taken, where h' is the water level in the control well (expressed in the same coordinate system as h), neglecting the impact of slopes in the pipelines. In drainage, water flows from the field to the well, $S \geq 0$. The drainage equation leads to a second order polynomial expression with coefficients a and b (the expressions for a and b are readily available from the formulae):

$$S(t) = \frac{A}{L^2}\left(4K\tilde{h}^2(t) + 8Kd_e\tilde{h}(t)\right) = a\tilde{h}(t) + b\tilde{h}^2(t) \tag{5}$$

In subsurface irrigation, the direction of the flow is reversed, $S < 0$, and it is assumed that the Hooghoudt equation applies to this direction also. A more appropriate form for the equation, allowing both drainage and irrigation is then

$$S(t) = sign\left(\tilde{h}(t)\right)\left[a\left|\tilde{h}(t)\right| + b\tilde{h}^2(t)\right] \tag{6}$$

and the ground water height dynamics are given by (3).

It is clear that the ground water table has an upper bound at $h \leq d$. In addition, the recharge is assumed irreversible, $R \geq 0$, within the span of dynamics considered. Many practical tunings can be considered to account for the simplifications in the model construction, e.g. by constraining maximum flows through pipelines $|S| < S^{max}$, flow direction dependent adjustements, etc.

2.4 Controlled subsurface drainage well

The water from subsurface drainage pipes is collected and flows to a well. The well has a controlled outflow. Supposing n alike subsurface pipe branches, the mass balance for the well is composed of the sum of all suction pipe flows T [m^3/s]:

$$T(t) = \sum_{i=1}^{n} S_i(t) \tag{7}$$

Denote the subsurface irrigation water, distributed via the well and the subsurface pipes, by U [m^3/s]. The mass balance is given by:

$$\rho A' \frac{dh'}{dt}(t) = \rho\left(T(t) - Y(t) + U(t)\right)$$

A' [m^2] and h' [m] denote the cross section area and height of the well. The outflow Y [m^3/s] is controlled by a valve, modelled as

$$Y(t) = CA_p\sqrt{2gh'(t)}$$

where C is the discharge coefficient [-], A_p [m^2] the outflow pipe cross section area and g [m/s^2] is the acceleration due to gravity. The model is then

$$\frac{dh'}{dt}(t) = \frac{1}{A'}\left[T(t) - u_C(t)\,CA_p\sqrt{2gh'(t)} + U(t)\right] \tag{8}$$

where u_C [%/100] denotes the opening of the control valve and $U(t) = u_I(t)U_{max}$ [kg/s] is the flow of the irrigation water to the well.

The level of the water in the controlled subsurface drainage well is constrained from bottom and top, $h'_{min} \leq h' \leq h'_{max}$, determined in the same coordinate system as h.

2.5 System model

The overall model is given by equations (1)–(8). Eq. (1) determines the recharge input from other rainfall. The model consists of three ode's: (2), (3) and (8), for system states $R(t)$, $h(t)$, and $h'(t)$, respectively. Eq. (4) defines the water head, eq. (5) the coefficients needed in the static Hooghoudt equation (6). In the envisioned simulations, the system inputs P, E, V and control actions u_C and u_I would determine the dynamic evolution of states.

Many of the coefficients can be approximated from subsurface drainage network geometry (A, L, n, d), well geometry $(A', A_p, h'_{min}, h'_{max})$, the joint valve and pipe properties (C), and basic physics (ρ, g). The field specific coefficients K and ϵ, must be chosen based on the soil properties. For d_e, approximative formula are available.

Assume that the drainage system is composed of n similar suction pipes. In state-space form, the system is then given by

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t))\,;\mathbf{y}(t) = g(\mathbf{x}(t))$$

where

$$\mathbf{x} = [R, h, h']^{\mathrm{T}}\,;\mathbf{u} = [u_C, u_I]^{\mathrm{T}}$$

subject to $h(t) \leq d$, $h'_{min} \leq h'(t) \leq h'_{max}$ and $R(t) \geq 0$. The irrigation is expressed as a function of the control opening u_I [-]. Typical measurements include $h'(t)$, in some setups $h(t)$, or both. In the numerical implementation, the time was expressed in days [d] and recharge, precipitation and irrigation intensities in mm per day [mm/d], and water levels in meters [m], providing both convienience in interpretation and better numerical conditioning.

3. PREDICTIVE CONTROL

3.1 Model predictive control under constraints

The control problem can be formulated as consisting of controlling the ground water table height to a desired level, subject to various constraints and uncertainties. In a possible scenario, it may be desired to ensure optimal growth whilst keeping the ground water level as high as possible to avoid release of greenhouse gas emissions. The soil bearing capacity also needs to be ensured, e.g., at times of soil preparation, sowing and harvesting. Also overflooding at times of heavy rains should be avoided. These requirements lead to a need to actively control the water table height. As the dominating dynamics of the system are slow, a basic error feedback solution is not feasible but a predictive (MPC) approach is highly preferred.

Note that many other meaningful formulations can be considered in the MPC context, e.g., looking at water table height as an interval constraint (tube) and focusing on minimization of waste of water, consumption of energy, etc. In what follows, the ground water table height is considered, for simplicity.

The control problem can be posed as a discrete-time open-loop optimization problem, where the cost J is minimized subject to constraints. Both cost and constraints, in general, are functions of future states $(k+1, k+2, ...)$, $t = kT_s$, T_s being the samping time, and control actions $(k, k+1, ...)$ in the future horizon. The optimization problem is then solved leading to an optimal sequence of actions. In MPC, the receeding horizon principle is applied, and the actions associated with the current instant k are applied in real time. At next time instant, the whole optimization procedure is repeated. In order to evaluate future values of J and constraints, model predictions are required, hence the name of the MPC method.

As a cost function, consider, for example, the groundwater level setpoint, h_{sp}, control with penalties on water losses due to drainage and irrigation costs:

$$J = c_0 \sqrt{\sum_{i=1}^{H_p} \left(h_{sp}(k+i) - h(k+i)\right)^2} + \sum_{j=\{C,I\}} c_j \sum_{i=1}^{H_p} |u_j(k+i)| \tag{9}$$

subject to upper and lower bound constraints on h, h', u_C, and u_I. The cost consists of a sum of squared deviations setpoint trajectory in the prediction horizon H_p and a term penalizing the magnitude of actions on well valve opening (costs due to lost water) and irrigation. Hard constraints on system inputs and outputs are due to system geometry, etc. The control horizon is implemented by limiting $u_j(k+i) = u(k+H_c-1)$ if $i \geq H_c$.

The weighing coefficients c_j, $j = \{0, C, I\}$, might be time varying as well, $c_j(k,i)$, to emphasize the importance of reaching the goals during certain periods, and/or weigh the increase in uncertainty in the future. It can also be useful to apply "blocking" Chen et al. (1995), i.e., to select the control interval as an integer multiple of the sampling interval, to reduce the unnnecessary degrees of freedom and computational load in optimization. For simplicity of notation, these are excluded from (9).

MPC is developed in discrete time, $t = kT_s$, which is natural for most computerized measurement, control and automation systems. In some cases it can be convienient to re-write the continuous-time model into it's discete-time counterpart. In general, the continuous-time model can always be solved numerically (using ode-solvers) one sampling period (T_s) ahead in time, thereby leading to a pseudo-discretized version. The latter approach was used in the simulations that follow, the optimization problem was solved using Matlab `fmincon`.

3.2 Feedback and uncertainties

MPC provides open-loop optimality. In general, the feedback in MPC comes via the state. If and when there are deviations between the real plant behaviour and the model predictions, the on-line measurements can be used to update the system state. If states can be directly measured, they may be used as estimates of the states. In general, state estimation is used, such as bayesian filtering (e.g. Kalman filtering) or deterministic observer design. Closed-loop optimality can be provided by (approximate) dynamic programming approaches (see e.g. Ikonen et al. (2016)), but their implementation easily becomes computationally intractable, especially since in this type of applications significant uncertainties reside in the future weather forecasts and the horizons are relatively long.

4. EXPERIMENTAL

4.1 Modeling

The system model was tuned for the Isosuo (Tyrnävä, Finland) case, for which 5 min data on well height (h') and precipitation ([mm]) from 3 June to 9 September 2021 was available. Precipitation was cumulated for each hour, and converted into precipitation intensity (P [mm/d]) Data for solar radiation (SR [W/m^2]) was available, which was used to approximate the evapotranspiration intensity ($E = 24 \times 0.00035 SR$ [mm/d]), thereby providing an estimate of precipitation entering the ground water ($Q = max(0, P - E)$ [mm/d]). The data contained no irrigation or controlled subsurface drainage actions, but the summer was exceptionally dry.

Parameters were roughly tuned by hand, adjusting the initial state in early June $x(0) = [17\text{mm/d}, 1.05\text{m}, 1.05\text{m}]$, discharge coefficient $u_C = 0.35$, and $\epsilon = 0.05$. Note, that the subsurface drain valve was closed during the summer, so that $u_C > 0$ indicates seepage or other consumption of water. The well water level measurement was not reliable for depths below 0.65 m. A simulation is illustrated in Fig. 2, indicating that higher frequency components of the model provide some correspondence to observed reality.

4.2 MPC

As a starting point, control with a perfect model and full information (i.e. perfect precipitation forecasts and full state measurements) is considered. A MPC is defined using the sum of squared deviation from setpoint in ground water table as the cost to minimize, see (9). The rainfall of the Isosuo case was used in the simulation, altering the desired ground water table hight between 0.9, 0.6 and 0.5 m. A sampling rate of 1 h was used throughout the simulations.

Simulations with various parameter and simulation settings are depicted in Figs 3–6. Impact of control horizon,

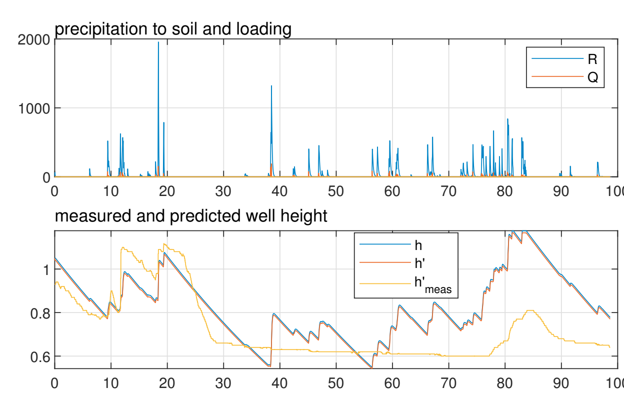

Fig. 2. Precipitation at Isosuo (top plot) and model predictions on groundwater table height with corresponding measurements. The performance at important frequencies is sufficient. Sampling time 5 min.

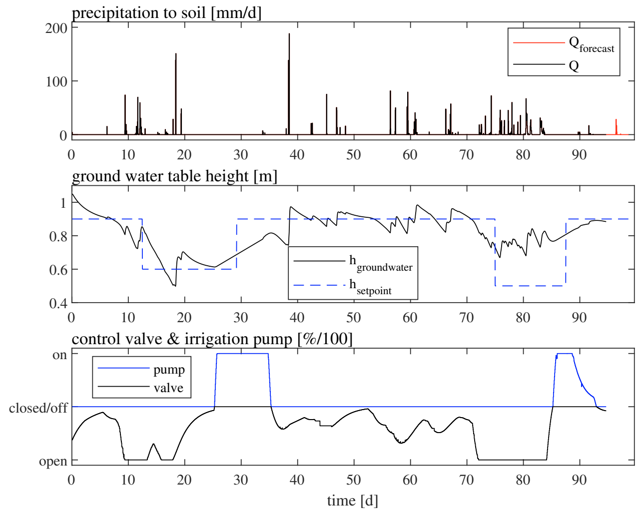

Fig. 3. MPC with basic setting: prediction horizon $H_p = 4$ days (96 samples), control horizon $H_c = 1$, $c_0 = 1$ and $c_C = c_I = 0$. Sampling time 1 hour.

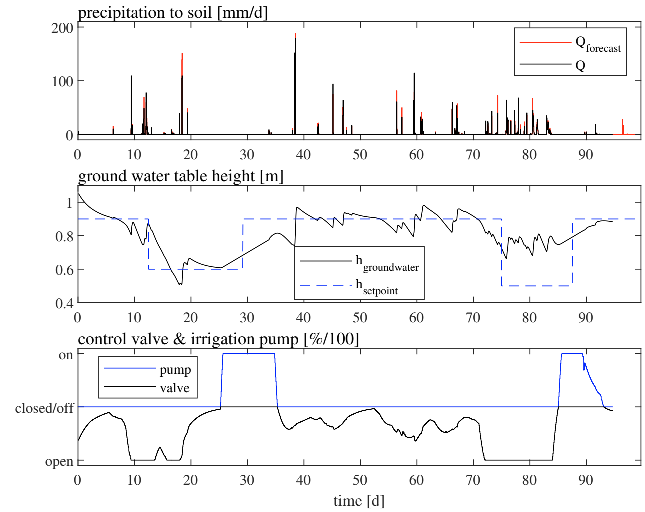

Fig. 5. MPC can cope with random ±50 % deviation between precipitation forecasts (red) and true precipitation (black).

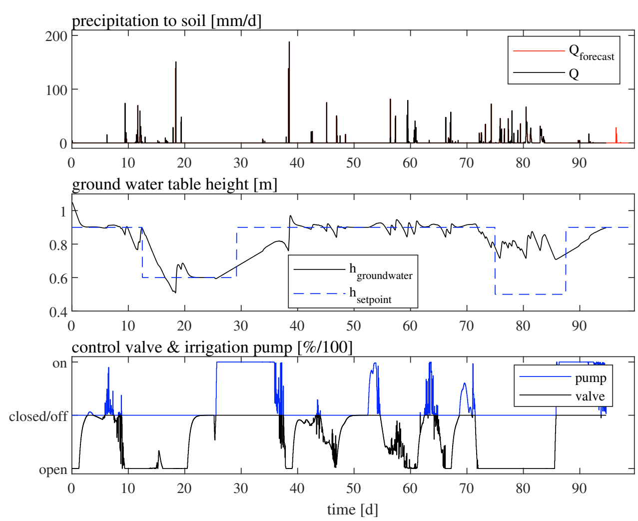

Fig. 4. The MPC control actions are more aggressive with $H_c = 2$. Blocking of 12 samples (see text and Fig. 7) was used in optimization.

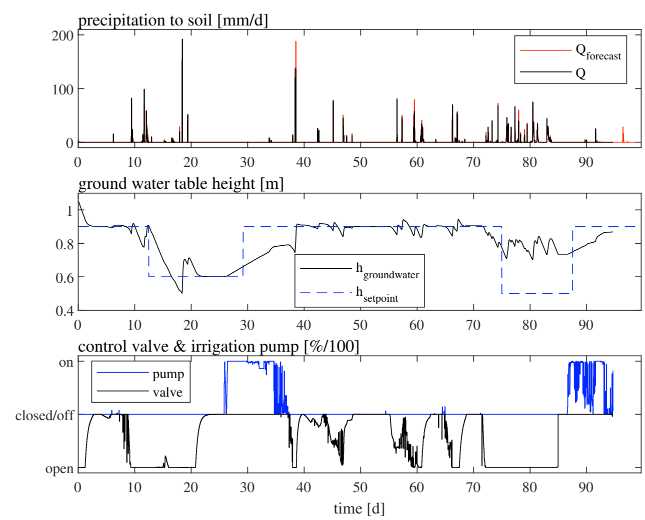

Fig. 6. MPC ($H_c = 2$) with costs on irrigation reduces the amount of water used for subsurface irrigation, $c_I = 0.03$.

costs on irrigation, and uncertainty in predictions are illustrated. As an important parameter, the prediction horizon was chosen to be 4 days in all simulations, but could have been altered as well, allowing e.g. the control system to prepare for future rainfalls/dry periods earlier. The validity of longer prediction horizons is obviously limited by the reliability of weather forecasts. Longer horizons imply significantly longer computing times, but for the problem at hand this poses no real restrictions. Due to constraints of space, these, and many more options, are not illustrated. Some practical notes are given at the end of the subsection.

The controller works as expected, see Fig. 3. First, the setpoint is at 0.9 m, which is reached by adjusting the well output valve. The change in the setpoint to 0.6 (at apprx 11 days), as well as the rainfalls predicted for the same period, are anticipated already some days before, and the well valve is fully open already at day 9. The ground water level drops to 0.6 m, the level actually drops below 0.6 in anticipation of the predicted rainfalls at day 18. When a step back to 0.9 m is made, there is no rain at sight, and irrigation is used to rise the groundwater table. Once the level is reached, the output valve is used to cope with rainfalls and keep the level at 0.9. When the setpoint is changed to 0.5 m (at day 75), quite heavy rainfalls take place. The valve is fully open, but the drainage capacity is not sufficient to drop the level to reach the setpoint. Finally, irrigation is again used to rise the water level from apprx. 0.7 to 0.9 m.

As mean level type of control ($H_c = 1$, long prediction horizon) is used, the control actions are smooth but the level is not tightly controlled to the setpoint. With $H_c = 1$ the optimization routine has only one control move to optimize the trajectory in the prediction horizon. Fig. 4 illustrates the case for $H_c = 2$. The use of both drainage

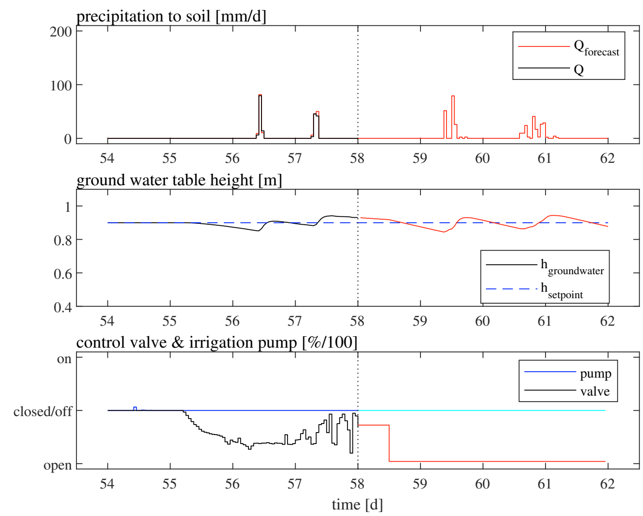

Fig. 7. A snapshot of MPC in decision making support at day 58. The vertical dashed line shows the current time. Past history, including deviations between real and forecasted precipitation are shown to the left. The predicted behaviour under proposed control ($H_c = 2$, blocking of 12 h) is shown to the right.

valve and irrigation is much more aggressive. The tighter setpoint control is well visible, e.g., during days 40–70.

Figure 5 illustrates the performance of MPC under uncertainties. The model and measurements were assumed perfect, but there are ±50 % random multiplicative deviations in the true rainfalls, compared to forecasts. The control system (with perfect state estimation) appears to tolerate well the errors in forecasts.

In a final simulation example, Fig. 6, a cost on the irrigation was included together with ±50 % errors in precipitation forecasts. Even with $H_c = 2$, the irrigation option was much less used (compare with Fig. 4, e.g., during days 40–70).

Comment on practical applications The sampling rate was chosen as 1 hour in the above simulations, with the underlying assumption of automatic control. Water is a sensitive parameter for farmers, however, and it may well be that in a practical control system all control actions need to be approved by the users, leading to a control interval of 12 or 24 hours. In this case also the monitoring and decision making support become pronounced, as there will be a human in the loop. The decision making of the field operator can be supported, e.g., by opening him/her the simulations behind the optimization, for example the expected behaviour of the water balance in the prediction horizon under the future scenario. This is illustrated in Fig 7. In a more active version, an option of what-if simulations may be provided, so as to pick a control action of preference taking into account all additional information available to the operator.

5. CONCLUSION

Groundwater level model predictive control was considered, proposing a simple model for soil water balance dynamics. The motivations for controlling the groundwater level are many, ranging from ensuring favorable growth of plants to scheduling of farming operations, efficient usage of water, and reduction of greenhouse gas emissions and water loadings. Model-based controlled subsurface drainage provides options for management of water balance, whether automatic or supporting human decision making.

The main contribution of this paper was to present and illustrate the concept of model-based control for groundwater table height control. More work is needed on making the modelling process straightforward and automatic, so as to cover feasibly the main dynamics of various fields differing in geometry, soil, crop, etc. Our on-going and future work focuses on i) improved dynamic modeling and data assimilation of the field water balance ii) feasible water balance control problem formulation, in terms of cost function, constraints, etc., iii) fusing the work on control automation with developments in the subsurface drainage control well infrastructure, and iv) gaining practical experiences from end-users and experimental peatland agriculture.

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