IQ Imbalance Compensation with a Pilot Sequence

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***Abstract*—Signal degradation caused by receiver in-phase/quadrature (IQ) processing branch imbalance (IQI) is known to increase bit error rates, and deteriorate both angle of arrival (AoA) and ranging estimation accuracies. In this paper, we present an IQI compensation procedure that leverages a pilot sequence to propose an IQI compensation method, which tolerates time synchronization errors. We then explore a single anchor positioning problem and show that the proposed procedure is effective in improving the position estimation accuracy. We evaluate its performance via computer simulations. The results show that the scheme outperforms an earlier method, which is blind in the sense that it does not capitalize the pilot sequence availability.**

***Index Terms*—IQ imbalance, pilot sequence, positioning, radio transceivers, channel estimation.**

I. Introduction

As part of 5G and the evolving 6G wireless systems, the requirement for accurate positioning in addition to the enhanced connectivity is becoming a high priority with the appearance of mmWave and THz band connectivity. The narrow beams enabled by the high center frequencies both require and enable accurate receiver and transmitter location estimates. Concurrently, the scaling of massive multiple-input multiple-output (MIMO) demands increasingly miniaturized antennas and radio frequency (RF) chain components at a low cost. Additionally, positioning enhancing and reduced capability (RedCap) device positioning are currently very active research areas for next generation network architectures [1]. Particularly, homodyne receivers also known as zero intermediate frequency (IF) receivers have received substantial attention, because they do not require IF filters. However, they must perform in-phase/quadrature (IQ) downconversion from the radio frequency (RF) to baseband. The process is more susceptible to IQ imbalance (IQI).

IQI has been shown to affect negatively the performance of orthogonal frequency division multiplexing (OFDM) systems [2] and to increase the position estimator's variance lower bound [3]. To tackle these issues, Tubbax *et al.* [4] propose a decision-directed approach in an OFDM context. Schuchert *et al.* [5] use the least mean squares (LMS) algorithm for OFDM IQI compensation. Tsui and Lin [6] attempt to jointly estimate and minimize transmitter and receiver IQ imbalance with a decision-directed LMS based approach. Maltera and Sterle [7] derive the maximum likelihood (ML) optimum blind estimator for a quadrature amplitude modulation (QAM) transmission and for a circularly symmetric Gaussian signal, and Chen *et al.* [8] use a widely-linear (WL) estimator for channel estimation and data detection while requiring either the channel covariance and complementary covariance or the IQI coefficients themselves.

Decision directed approaches for IQI compensation require symbol synchronization. They can also suffer from performance degrations due to erroneous decisions. Therefore, the decision based schemes become ineffective if the IQI strongly degrades the signal quality [4]. Furthermore, blind estimates make assumptions about the data distribution which may not be ultimately accurate, compromising the quality of the estimates. What is more, the blind estimates do not take advantage of the availability of pilot or training sequences in practical wireless standards.

In this paper, we explore the IQI compensation with a pilot sequence. The target is to derive an efficient compensation scheme capitalizing the available pilot signals. The approach utilizes the channel estimates computed in the receiver. We apply the scheme to positioning or more precisely ranging. The numerical results show that we can achieve better IQI compensation compared to the earlier blind approach [7]. The performance benefit is possible by incorporating information from a pilot sequence and a channel estimate to estimate the IQI parameters. This comes at a moderate computational cost of channel estimation, which, however, is usually needed in the receiver anyways. We will demonstrate that the deteriorating effects of IQI on positioning estimation have been effectively compensated, while outperforming a Gaussian ML blind estimator.

The rest of the paper is organized as follows. In Section II, we present the system model detailing the channel and IQI models. in Section III, we expand on the equivalent channel estimation method that will be used in the proposed IQI compensation method. Section IV derives the algorithm itself. In Section V, we present numerical results outlining the effectiveness our procedure in a positioning scenario. Finally, Section VI concludes the paper.

II. System Model

Consider the case of a single receiver and transmitter with a line of sight (LOS) link. Let $\mathbf{s}(t)$ be the baseband equivalent transmitted signal at time t. The baseband equivalent received signal before downconversion is given by

$$\begin{aligned}\mathbf{x}(t) &= \gamma \mathbf{a}_R(\phi_R)\mathbf{a}_T^H(\phi_T)\mathbf{s}(t-\tau) + \mathbf{w}(t) \\ &= \mathbf{H}\mathbf{s}(t-\tau) + \mathbf{w}(t),\end{aligned} \tag{1}$$

where γ is the complex path gain, $\mathbf{a}_R(\phi_R)$ and $\mathbf{a}_T(\phi_T)$ are, respectively, the receiver and transmitter array response vectors as a function of the angles of arrival and departure, τ is the propagation delay from the transmitter to the receiver, and $\mathbf{w}(t)$ is complex circularly symmetric additive white Gaussian noise (AWGN). The channel matrix $\mathbf{H}$ is $N_R \times N_T$ where N_R and N_T denote the number of receive and transmit antennas, respectively.

In a positioning context, position estimates rely on extracting the angle of arrival or departure and the distance or ranging measurements from the received signal. The angle of arrival estimates are usually performed using an algorithm such as multiple signal classification (MUSIC) [9] or estimation of signal parameters via rotational invariance (ESPRIT) [10]. In the LOS case, ranging is usually done by measuring the time of flight (ToF) τ.

Non-idealities of the in-phase and quadrature components of the local oscillator (LO) signal of the converter (up or down converter) corrupt the received signal and deteriorate the quality of those estimates giving rise to IQI. From an economic and practical standpoint, it is convenient to mitigate these effects in software or baseband processing, specially concerning RedCap devices and transceivers with cheaper hardware. Let $\mathbf{\Theta}_A$ and $\mathbf{\Theta}_B$ denote the receiver IQI matrices, $\mathbf{\Theta}_A = \mathrm{diag}(\alpha_1, \ldots, \alpha_{N_R})$ and $\mathbf{\Theta}_B = \mathrm{diag}(\beta_1, \ldots, \beta_{N_R})$. These matrices allow us model imbalanced IQ demodulation at each RF chain [11]. The IQI corrupted received signal is computed as

$$\mathbf{y} = \mathbf{\Theta}_A\mathbf{x} + \mathbf{\Theta}_B\mathbf{x}^* = \mathbf{\Theta}\mathbf{x}_e, \tag{2}$$

where $\mathbf{x}^*$ is the complex conjugate of $\mathbf{x}$, $\mathbf{\Theta} = \begin{bmatrix}\mathbf{\Theta}_A & \mathbf{\Theta}_B\end{bmatrix}$, $\mathbf{x}_e = \begin{bmatrix}\mathbf{x}^T & \mathbf{x}^H\end{bmatrix}^T$, and

$$\alpha_r \triangleq \frac{(1 + m_r e^{-j\psi_r})}{2} \tag{3}$$

$$\beta_r \triangleq \frac{(1 - m_r e^{j\psi_r})}{2}, \tag{4}$$

with $m_r \triangleq 1 + \epsilon_r$, where ϵ_r and ψ_r are real numbers called the receiver amplitude and phase IQI parameters. The receiver IQ demodulation under IQ imbalance is represented in Fig. 1.

In (1), $\mathbf{x}(t)$ is the baseband equivalent of $\mathbf{x}^{RF}(t)$. The way that the equations have been defined allows us to bypass the upconversion and downconversion operations and focus only in the IQI effects. However, in a more detailed graphical representation of the IQ demodulation such as the one in Fig. 1, $\mathbf{x}(t)$ must be distinguished from $\mathbf{x}^{RF}(t)$ which is the signal that would be physically received at the antenna.

Assume that the channel propagation delay is well approximated by an integer number of samples. Then each transmitted pilot sequence sample $\mathbf{s}(i)$, where i denotes the discrete time

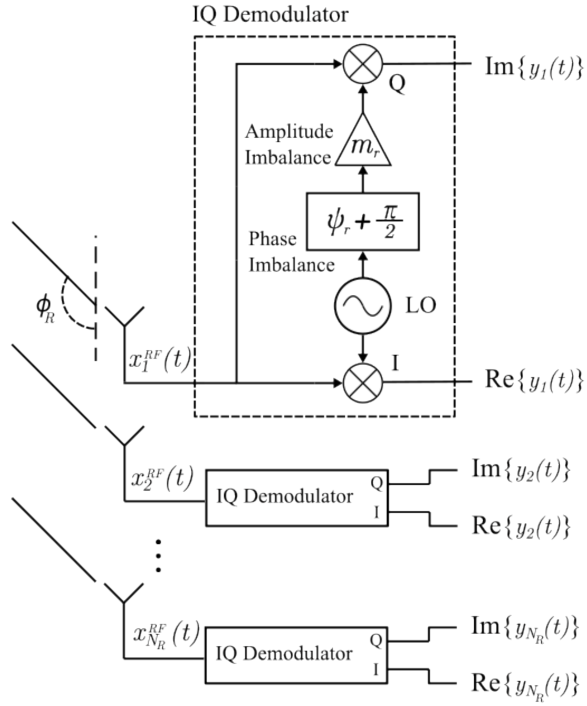

Fig. 1. Receiver IQ demodulation scheme under IQ imbalance. Inside the IQ demodulator, the triangle block denotes an amplitude gain and the square block denotes a phase increment.

sample index, has a corresponding received signal sample $\mathbf{y}(i)$ at the appropriate sampling time accounting for channel propagation time following the relation

$$\mathbf{y}(i) = \mathbf{\Theta}\mathbf{H}_e\mathbf{s}_e(i) + \mathbf{n}(i) = \mathbf{G}\mathbf{s}_e(i) + \mathbf{n}(i), \tag{5}$$

where $\mathbf{s}_e^T(i) = \begin{bmatrix}\mathbf{s}(i) & \mathbf{s}^H(i)\end{bmatrix}^T$, $\mathbf{H}_e$ is block diagonal of the form $\mathrm{diag}(\mathbf{H}, \mathbf{H}^*)$, and $\mathbf{n} = \mathbf{\Theta}_A\mathbf{w} + \mathbf{\Theta}_B\mathbf{w}^*$. From (5) we can write

$$\mathbf{Y} = \begin{bmatrix}\mathbf{y}(0) & \cdots & \mathbf{y}(N_s - 1)\end{bmatrix} = \mathbf{\Theta}\mathbf{H}_e\mathbf{S}_e + \mathbf{N} \tag{6}$$

$$\mathbf{S}_e = \begin{bmatrix}\mathbf{s}_e(0) & \cdots & \mathbf{s}_e(N_s - 1)\end{bmatrix} \tag{7}$$

$$\mathbf{N} = \begin{bmatrix}\mathbf{n}(0) & \cdots & \mathbf{n}(N_s - 1)\end{bmatrix}. \tag{8}$$

Manipulating this further, using the identity

$$\mathrm{vec}(\mathbf{\Theta}\mathbf{H}_e\mathbf{S}_e) = (\mathbf{S}_e^T \otimes \mathbf{\Theta})\mathrm{vec}(\mathbf{H}_e) \tag{9}$$

we get $\mathbf{S}_e^T \otimes \mathbf{\Theta} = \breve{\mathbf{S}}_e^T\breve{\mathbf{\Theta}}$, where $\breve{\mathbf{S}}_e = \mathbf{S}_e \otimes \mathbf{I}_{N_R}$ and $\breve{\mathbf{\Theta}} = \mathbf{I}_{2N_T} \otimes \mathbf{\Theta}$. This yields

$$\breve{\mathbf{y}} = \breve{\mathbf{S}}_e^T\breve{\mathbf{\Theta}}\breve{\mathbf{h}}_e + \breve{\mathbf{n}} = \breve{\mathbf{S}}_e^T\breve{\mathbf{g}} + \breve{\mathbf{n}}, \tag{10}$$

where $\breve{\mathbf{h}}_e = \mathrm{vec}(\mathbf{H}_e)$ and $\breve{\mathbf{n}} = \mathrm{vec}(\mathbf{N})$. We call $\breve{\mathbf{g}} = \breve{\mathbf{\Theta}}\breve{\mathbf{h}}_e$ the vectorized *equivalent channel*. By writing the equivalent channel in this vectorized form, we can express the covariance and complementary covariance matrix estimates derived from (10) as

$$\mathbf{R}_{\breve{\mathbf{g}}_{\text{est}}} = (\breve{\mathbf{S}}_e^T)^\dagger(\mathbb{E}\left\{\breve{\mathbf{y}}\breve{\mathbf{y}}^H\right\} - \mathbf{R}_{\breve{\mathbf{n}}})(\breve{\mathbf{S}}_e^*)^\dagger \tag{11}$$

$$\mathbf{Q}_{\breve{\mathbf{g}}_{\text{est}}} = (\breve{\mathbf{S}}_e^T)^\dagger(\mathbb{E}\left\{\breve{\mathbf{y}}\breve{\mathbf{y}}^H\right\} - \mathbf{Q}_{\breve{\mathbf{n}}})(\breve{\mathbf{S}}_e)^\dagger \tag{12}$$

where $\dagger$ denotes the pseudoinverse.

III. Equivalent Channel Estimation

It is not generally possible to leverage the information from a known pilot sequence for improved IQI coefficient estimator performance unless the channel is precisely estimated, otherwise we cannot separate the channel effects and the IQI effects on the pilot sequence. Also the channel cannot be estimated independently from the IQI, because every received sample is only observed after IQI. Since IQI is essentially widely-linear, to properly capture the effects of IQI in the received sequence it is necessary to estimate the channel as a widely-linear operator.

For brevity, we will refer to the vectorized channel and augmented channel simply as the channel and augmented channel, respectively. Assume that the vectorized equivalent channel $\breve{\mathbf{g}}$ is constant and we only observe one single realization during the transmission of the whole sequence. We will now perform the augmented eigenvalue decomposition (AEVD) of the sample channel augmented covariance matrix as described in [12] and use it to obtain an approximation for the equivalent channel. The following procedure with the AEVD allows us to find the vector that has the desired covariance and complementary covariance matrices. Let $\mathbf{x}$ be an arbitrary complex zero mean random vector in $\mathbb{C}^n$, and let $\mathbf{x} = \mathbf{a} + j\mathbf{b}$ and $\mathbf{z} = \left[\, \mathbf{a}^T \; \mathbf{b}^T \,\right]^T$. Also define

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & j\mathbf{I} \\ \mathbf{I} & -j\mathbf{I} \end{bmatrix} \tag{13}$$

We write the EVD of $\mathbf{R_z} = \mathbb{E}\left\{\mathbf{z}\mathbf{z}^T\right\}$ as

$$\mathbf{R_z} = \mathbf{U} \begin{bmatrix} \frac{1}{2}\boldsymbol{\Xi}_1 & \mathbf{0} \\ \mathbf{0} & \frac{1}{2}\boldsymbol{\Xi}_2 \end{bmatrix} \mathbf{U}^H, \tag{14}$$

where the eigenvalues are organized in an descending order and $\boldsymbol{\Xi}_1$ and $\boldsymbol{\Xi}_2$ are the diagonal matrices with the odd and even eigenvalues. From (14) and the relation $\mathbf{R}_{e,\mathbf{x}} = \mathbf{T}\mathbf{R_z}\mathbf{T}^H$ we get

$$\mathbf{R}_{e,\mathbf{x}} = \begin{bmatrix} \mathbf{R_x} & \mathbf{Q_x} \\ \mathbf{Q_x^*} & \mathbf{R_x^*} \end{bmatrix} = \mathbf{V}\boldsymbol{\Lambda}_e\mathbf{V}^H \tag{15}$$

$$\mathbf{V} = \left(\frac{1}{2}\mathbf{T}\mathbf{U}\mathbf{T}^H\right) = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \\ \mathbf{V}_2^* & \mathbf{V}_1^* \end{bmatrix} \tag{16}$$

$$\boldsymbol{\Lambda}_e = \frac{1}{2}\begin{bmatrix} \boldsymbol{\Xi}_1 + \boldsymbol{\Xi}_2 & \boldsymbol{\Xi}_1 - \boldsymbol{\Xi}_2 \\ \boldsymbol{\Xi}_1 - \boldsymbol{\Xi}_2 & \boldsymbol{\Xi}_1 + \boldsymbol{\Xi}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}_1 & \boldsymbol{\Lambda}_2 \\ \boldsymbol{\Lambda}_2 & \boldsymbol{\Lambda}_1 \end{bmatrix}, \tag{17}$$

where $\mathbf{R_x} = \mathbb{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$ and $\mathbf{Q_x} = \mathbb{E}\left\{\mathbf{x}\mathbf{x}^T\right\}$

Let us rewrite $\tilde{\mathbf{R}}_{e,\mathbf{x}} = \mathbf{V}\boldsymbol{\Lambda}_e\mathbf{V}^H$ by explicitly carrying out the block matrix products. For our purposes, we only care about the top left block. Suppose now that all the eigenvalues besides λ_1 are zero. In that case, $\boldsymbol{\Lambda}_1 = \boldsymbol{\Lambda}_2 = \lambda_1\mathbf{I}$, and the top block, which corresponds to the covariance matrix of $\mathbf{x}$, is equal to $\lambda_1(\mathbf{v}_1\mathbf{v}_1^H + \mathbf{v}_2\mathbf{v}_1^H + \mathbf{v}_1\mathbf{v}_2^H + \mathbf{v}_2\mathbf{v}_2^H)$. We look for a vector $\breve{\mathbf{g}}$ such that $\breve{\mathbf{g}}\breve{\mathbf{g}}^H = \lambda_1(\mathbf{v}_1\mathbf{v}_1^H + \mathbf{v}_2\mathbf{v}_1^H + \mathbf{v}_1\mathbf{v}_2^H + \mathbf{v}_2\mathbf{v}_2^H)$. It is easy to see that $\breve{\mathbf{g}} = \pm\sqrt{\lambda_1}(\mathbf{v}_1 + \mathbf{v}_2)$ satisfies this requirement and is unique up to the sign. In this framework, $\breve{\mathbf{g}}$ is already constrained to satisfy $\breve{\mathbf{g}}\breve{\mathbf{g}}^T = \tilde{\mathbf{Q}}_{\breve{\mathbf{g}}}$. We only need now to estimate if the sign is positive or negative. Since we are using a pilot sequence, one possible method is to compare the squared error of the received signal and the predicted received signal using the channel as in

$$\sum_{i=1}^{N_s} \|\tilde{\mathbf{G}}\mathbf{s}_e(i) - \mathbf{y}(i+\hat{k}_s)\|^2 \underset{\tilde{\breve{\mathbf{g}}}}{\overset{-\tilde{\breve{\mathbf{g}}}}{\gtrless}} \sum_{i=1}^{N_s} \|-\tilde{\mathbf{G}}\mathbf{s}_e(i) - \mathbf{y}(i+\hat{k}_s)\|^2, \tag{18}$$

where $\tilde{\mathbf{G}}$ is the channel in matrix form obtained from $\tilde{\breve{\mathbf{g}}}$ and $\hat{k}_s$ is a propagation delay estimate in samples. In other words, always opt for the sign that produces predictions with smaller sum of square errors.

IV. IQI Parameter and Signal Estimation

In this section, we derive the estimator of the IQI coefficients and system parameters. The derivations below are valid when RF chains share the same IQI coefficients. If the IQI characteristics of the chains are different, then the following procedure is equivalent to restricting the computations to the subsystem where the coefficients are the same.

Considering (1), (2), and (5), we can show that the covariance and complementary covariance of $\mathbf{y}$ are given by

$$\mathbf{R_y} = \boldsymbol{\Theta}\mathbf{H}_e\mathbb{E}\left\{\mathbf{s}_e\mathbf{s}_e^H\right\}\mathbf{H}_e^H\boldsymbol{\Theta}^H + \mathbf{R_n} \tag{19}$$

$$\mathbf{Q_y} = \boldsymbol{\Theta}\mathbf{H}_e\mathbb{E}\left\{\mathbf{s}_e\mathbf{s}_e^T\right\}\mathbf{H}_e^T\boldsymbol{\Theta}^T + \mathbf{Q_n}, \tag{20}$$

where $\mathbf{R_n}$ and $\mathbf{Q_n}$ are the covariance and complementary covariance matrices of $\mathbf{n}$, respectively. The expectations are known, because $\mathbf{s}$ is a deterministic pilot sequence

Suppose we already possess a perfect estimate of the equivalent channel $\mathbf{G} = \boldsymbol{\Theta}\mathbf{H}_e$, then we can get an estimate of $\mathbf{H}$ by first computing the block diagonal structure preserving least squares solution of

$$\breve{\mathbf{h}}_e^{LS}(\epsilon_r, \psi_r) = \underset{\breve{\mathbf{h}}_e'}{\operatorname{argmin}} \|\breve{\mathbf{g}} - \breve{\boldsymbol{\Theta}}(\epsilon_r, \psi_r)\breve{\mathbf{h}}_e'\|_2^2,$$

$$\text{s.t. } \breve{\mathbf{h}}_e' = 0 \text{ if it is a secondary diagonal block element} \tag{21}$$

This can be computed by translating the usual minimum norm solution along the null-space of $\breve{\boldsymbol{\Theta}}(\epsilon_r, \psi_r)$ to zero-out the desired $2N_rN_t$ elements. This solution does not exist when the null space of $\breve{\boldsymbol{\Theta}}$ does not span the secondary diagonal blocks. In that case the solution with nonzero left upper block and at most $2N_RN_T$ zeros in total must be used (such as the output of Matlab's *mldivide*). Then, omitting the dependence on ϵ_r and ψ_r to avoid heavy notation, $\tilde{\mathbf{H}}$ is the upper left $N_r \times N_t$ block of $\text{vec}^{-1}(\breve{\mathbf{h}}_e^{LS})$, and $\tilde{\mathbf{H}}_e = \text{Diag}(\tilde{\mathbf{H}}, \tilde{\mathbf{H}}^*)$.

Suppose we also know $\mathbf{R_n}$ and $\mathbf{Q_n}$ with sufficient precision. We thereby have candidate covariance and complementary covariance matrices of $\mathbf{y}$ as functions of ϵ_r and ψ_r

$$\tilde{\mathbf{R}}_\mathbf{y}(\epsilon_r, \psi_r) = \mathbf{G}'\mathbb{E}\left\{\mathbf{s}_e\mathbf{s}_e^H\right\}\mathbf{G}'^H + \mathbf{R_n} \tag{22}$$

$$\tilde{\mathbf{Q}}_\mathbf{y}(\epsilon_r, \psi_r) = \mathbf{G}'\mathbb{E}\left\{\mathbf{s}_e\mathbf{s}_e^T\right\}\mathbf{G}'^T + \mathbf{Q_n} \tag{23}$$

$$\mathbf{G}'(\epsilon_r, \psi_r) = \boldsymbol{\Theta}(\epsilon_r, \psi_r)\tilde{\mathbf{H}}_e(\epsilon_r, \psi_r). \tag{24}$$

If the pilot sequence is sufficiently long, the true covariance and complementary covariance matrices can be estimated with an arbitrary precision. Suppose we have a near perfect estimate of $\mathbf{R_y}$ and $\mathbf{R_y}$, then choose the candidate matrices that minimize the objective function

$$f(\epsilon_r, \psi_r) = \|\mathbf{E_R}(\epsilon_r, \psi_r)\|_F^2 + \|\mathbf{E_Q}(\epsilon_r, \psi_r)\|_F^2 \tag{25}$$
$$= \text{tr}\{\mathbf{E_R^H E_R}\} + \text{tr}\{\mathbf{E_Q^H E_Q}\}, \tag{26}$$

in which $\mathbf{E_R} = \mathbf{R_y} - \tilde{\mathbf{R}}_\mathbf{y}$ and $\mathbf{E_Q} = \mathbf{Q_y} - \tilde{\mathbf{Q}}_\mathbf{y}$. And finally $(\epsilon_r^{opt}, \psi_r^{opt}) = \text{argmin} f(\epsilon_r, \psi_r)$. The optimization problem is usually well-behaved and the optima are easy to find assuming the starting point is within typical values for the IQI parameters. An example of the objective function shape is shown in Fig. 2.

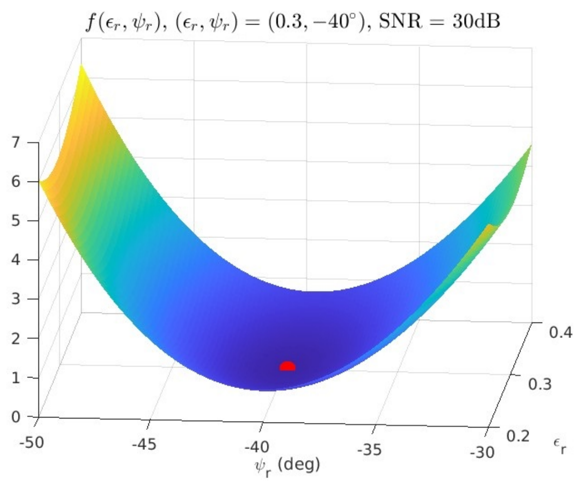

Fig. 2. Surface plot of $f(\epsilon_r, \psi_r)$ for an 8×2 Rayleigh channel at 30 dB SNR and a 1000 sample white Gaussian noise pilot sequence. The red dot shows the position of the optimal value.

We then use the estimated IQI parameters to get $\mathbf{x}$ from $\mathbf{y}$ using

$$\begin{bmatrix} \tilde{\mathbf{x}} \\ \tilde{\mathbf{x}}^* \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{\Theta}}_A & \tilde{\mathbf{\Theta}}_B \\ \tilde{\mathbf{\Theta}}_B^* & \tilde{\mathbf{\Theta}}_A^* \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} \tag{27}$$

V. Numerical Examples

The simulation setup used to evaluate the effectiveness of the proposed method is a single anchor position estimation problem using the first samples of a 5G NR random OFDM waveform as a pilot sequence. The used waveform is just one example to demonstrate the performance. Many other reasonable pilot sequences could be used in a straightforward manner. The simulations use the resources offered by the Matlab 5G NR package, and the fundamental simulation parameters are described in Table I. A 5G NR waveform with 52 resource blocks and 60 kHz subcarrier spacing has a 37.44 MHz bandwidth. The transmitter and receiver are coplanar with geometry and angles as defined in Fig. 3. In all results the receiver is located at (0,0) and the transmitter at (40,10) with an angle of $\phi_0 = \pi/2$.

Signal detection and synchronization is conducted by finding the maximum value of the output of a matched filter. This procedure ensures coarse timing and symbol synchronization if

TABLE I
Fundamental simulation parameters.

Channel Model	CDL
Path gain	0 dB
Subcarrier Spacing	60 kHz
Resource Blocks	52
Cyclic Prefix	Extended
Modulation	QPSK
Carrier Frequency	4 GHz
Tx Antennas	1
Rx Antennas	8
Channel Sample Density	*inf*
DoA Algorithm	MUSIC
Ranging Algorithm	Matched filter detector

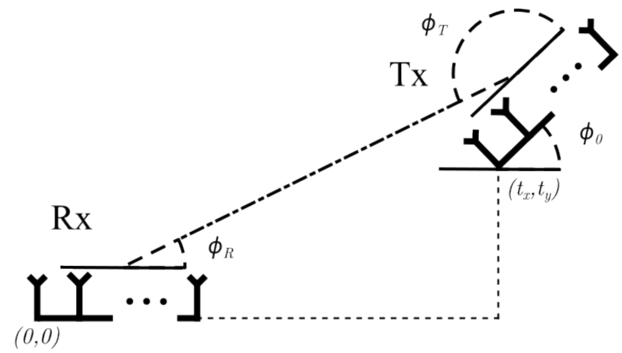

Fig. 3. Transmitter and receiver geometry.

the waveform is properly oversampled. Oversampling ensures that adjacent samples are highly correlated, thus making the channel estimation and IQI computations more robust to minor synchronization errors. We assume known transmission time and perfectly synchronized clocks between Tx and Rx as a way of isolating the impacts of IQI on ranging accuracy.

The equivalent channel estimation and the IQI parameter estimation procedures as described consider that the received signal samples $\mathbf{y}(i)$ are detected at the correct instant, i.e., at the true first signal sample. If this is satisfied, then the correct relation between the received sample $\mathbf{y}(i)$ and the transmitted sample of the pilot sequence $\mathbf{s}(i)$ can be established.

In general, the signal may not be detected at the perfect time instant, because in a practical situation it is difficult to determine the optimum sampling time and avoiding such a strict synchronization requirement simplifies the procedure. The equations for the maximum likelihood covariance and complementary covariance matrices of $\mathbf{y}$, will now incorporate this error in their calculation. The effects of this error are not noticeable at all in the root mean square error (RMSE) of the position estimates, to the point where results of the simulations with imperfect synchronization are indistinguishable from the perfectly synchronized case by simply visualizing the plots.

We consider that all RF chains share the same IQI coeffients in the simulations, this allows us to visualize the impact of the phase and amplitude IQI parameters in the mean square error (MSE) of the position estimates. We first present an IQI parameter grid sweep over (ϵ_r, ψ_r) in Figs. 4 and 5 for SNR

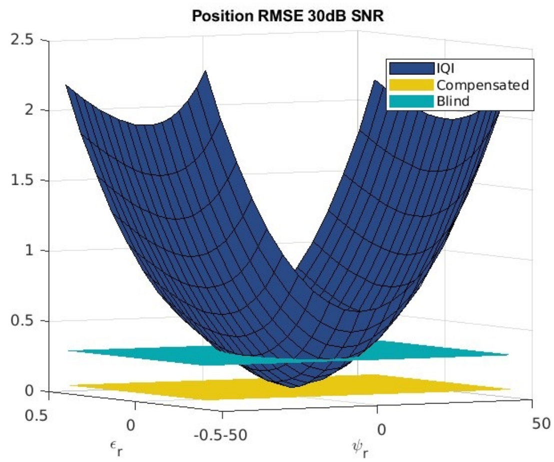

Fig. 4. Position RMSE grid sweep for the IQI affected signal, signal compensated signal with our method, and signal compensated with the blind estimator at 30 dB SNR.

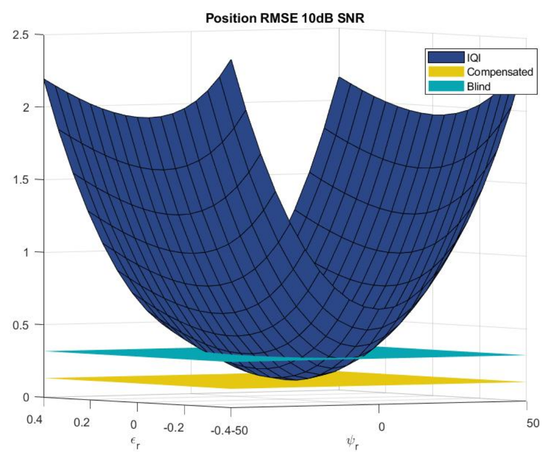

Fig. 5. Position RMSE grid sweep for the IQI affected signal, signal compensated signal with our method, and signal compensated with the blind estimator at 10 dB SNR.

values of 30 dB and 10 dB, respectively. The number of iterations per grid point is 50, the pilot sequence has 50 samples before interpolation with rate 16. The signal sampling rate is originally set by the OFDM modulator as a function of the bandwidth. We linearly interpolate the samples to increase the time resolution of the detector up to values that are acceptable in a ranging context, this is comparable to oversampling in a practical context. The method is very successful at eliminating the impact of IQI on positioning accuracy. By analysing Figs. 4 and 5 we can see that the compensated and the original signals without IQI produce positioning estimates of almost identical RMSE, i.e., the compensated RMSE is close to the plane tangent to the IQI RMSE surface at the $(\epsilon_r, \psi_r) = (0, 0)$ point and parallel to the $\epsilon_r\psi_r$-plane. This represents a major improvement in comparison to the RMSE of the positioning estimates extracted from the IQI corrupted signal. We also observe that the performance is almost the same at both SNR values, which is an interesting phenomenon and will be addressed below.

As a reference point, to verify the relative performance of the method presented in this paper, we compare it to a blind maximum likelihood estimator for the IQI parameters assuming Gaussian received signal, such as the one described in [7]. Modifications have been made to adapt the IQI model from the one used [7] to the one used here. In the described simulation scenario, and at 30 dB SNR, our model outperformed the blind estimator by an average 24.56 cm reduction in position RMSE.

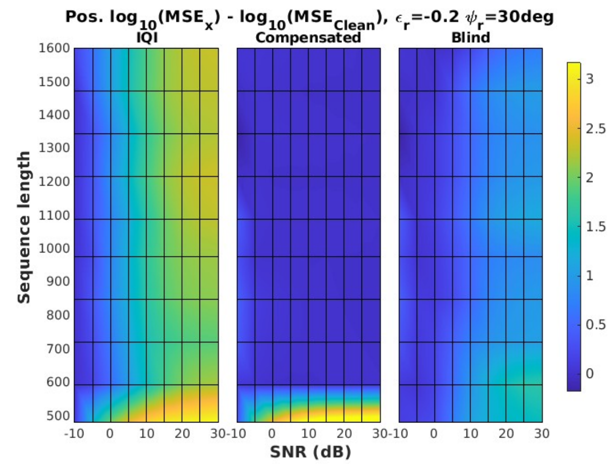

Fig. 6. Difference between the base 10 logarithms of the position estimate MSE of a particular case and the case without IQI (clean) as a function of the transmitted sequence length and SNR. IQI parameters set to $\epsilon_r = -0.2$ and $\psi_r = 30°$. The plots show from left to right: the uncompensated case (IQI), the case with the compensated signal using our method (Compensated), and the case where compensation is done with a blind estimator (Blind).

In Fig. 6 we assess the impacts of the SNR and the transmitted pilot sequence length on the estimated position MSE. We present the results as the MSE ratio in dB, i.e., the difference of the MSE logarithms of a particular case and the clean case. This keeps the values within a convenient range. Notice that once the pilot sequence is long enough (around 600 samples in this particular example) our method is capable of perfectly compensating the IQI impacts on the position estimate no matter the SNR. For negative SNR, the position estimate without IQI is already very inaccurate and IQI does not significantly worsen the estimates in this case, thus the values to the left approach 0. Another interesting effect is that the uncompensated position estimates somewhat improve for larger sequence length, this is largely because more samples allow a better pseudospectrum estimate in the MUSIC algorithm, used for DoA estimation. A longer pilot sequence also makes the matched filter include more samples, reducing the chance of spurious detections.

In Fig. 7 we present a comparison between the estimated position RMSE values achieved by our method (Compensated), the blind estimator (Blind), the uncompensated signal (IQI), and the case where no IQI exists (Clean). In this figure, the IQI coefficients are kept constant at $\epsilon_r = -0.4$ and $\psi_r = 30°$ and the results are averaged over 50 iterations. The

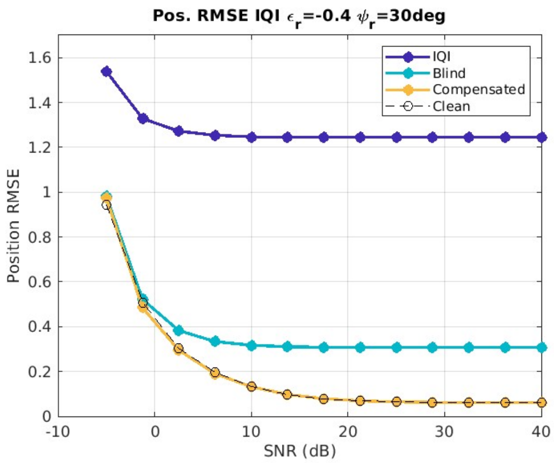

Fig. 7. Position estimate RMSE as a function of the SNR with fixed pilot sequence length.

pilot sequence length is also kept constant at 800 samples (50 samples interpolated at a rate of 16). We observe that our method almost completely eliminates the IQI for all SNR values, i.e., the Compensated and Clean curves basically overlap. Additionally, the proposed method clearly presents a major performance improvement when compared to the blind estimator or the uncompensated case.

It is also noticeable in Fig. 7 that the RMSE saturates at high SNR instead of improving abitrarily. This can be attributed to finite time resolution due to sampling rate, finite DoA angle resolution in the MUSIC algorithm, suboptimal pilot sequence choice, because the used waveform is not optimized for the application, and possibly short pilot sequence length. Exploring this SNR performance saturation in more detail is left for future work.

Comparing computational complexity and execution time between both methods, we have timed them on a simulation of the transmission of $N_s = 800$ samples, $N_r = 8$ receiver antennas, $N_t = 1$ transmit antennas, over a complex Rayleigh channel with $\text{Var}\{h_{ij}\} = 1$, averaged over 500 trials. The proposed method takes on average (excluding channel estimation) 22.363 ms of simulation time in a Linux server in an Intel Xeon Gold 6126 CPU, while the blind estimation takes 0.808 ms on the same hardware. Our method comes at the disadvantage of requiring channel estimation of a channel in widely-linear form. The proposed method, including channel estimation, took on average 1.170 seconds to compute. Naturally, the channel estimate can be used for other procedures that would also require a channel estimate. We want to emphasize that the presented times are computer simulation times used just for relative comparisons, and the actual computing time on dedicated hardware would be much faster. This ensures that IQI estimation and compensation do not affect the ranging performance at all.

VI. Conclusion

We have put forward a pilot sequence based IQI compensation procedure using channel and IQI parameter estimation. We have verified the impacts of IQI on position estimation accuracy and assessed the effectiveness and relative superior accuracy of our method in restoring positioning performance. We evaluated the performance as a function of the SNR and pilot sequence length. The results show that the proposed scheme vastly outperforms the earlier blind estimation method. The scheme has slightly higher computational complexity due to channel estimation, which often needed in the receiver in any case.

As future work, we have reason to believe that this procedure can readily adapted to be 5G NR conforming and provide superior IQI compensation performance compared to other OFDM based methods. This would be achieved by reformulating the expressions in an OFDM context and adapting the channel estimation procedure to the 5G standard. The performance would then be assessed by simulating the throughput and error rates (BER, SER, and FER). It is also of interest to jointly compensate transmitter and receiver IQI without relying on additional hardware.

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