

Analysis of PAPR in OTFS Modulation with Classical Selected Mapping Technique

Sneha Chennamsetty¹, Subbarao Boddu², Prabhu Chandhar³, Krishna Chaitanya Bulusu⁴

^{1,2} Department of Electrical and Electronics Engineering, Mahindra University, Hyderabad, India

³Chandhar Research Labs, Chennai, India

⁴Centre for Wireless Communications, University of Oulu, Finland

¹sneha20peee007@mahindrauniversity.edu.in, ²subbarao.boddu@mahindrauniversity.edu.in,

³prabhu@chandhar-labs.com, ⁴satish.bulusu@gmail.com

Abstract—Orthogonal Time Frequency Space (OTFS) is the advanced stage of modulation for this fast-growing generation. This is the waveform proposed to be used in next-generation communication systems i.e., 6G, as it has the advantage of high data rate, better flexibility, requires a less cyclic prefix and less amount of peak to average power ratio (PAPR) compared to Orthogonal Frequency Division Multiplexing (OFDM). To reduce the amount of PAPR in the OTFS, the best method is the classical selected mapping (SLM) technique. This paper presents the PAPR analysis in OTFS modulation by using classical SLM and the obtained results are compared against the OFDM modulation.

Index Terms—6G, OTFS, PAPR, SLM, OFDM.

I. INTRODUCTION

Multi-carrier modulation has drawn a lot of interest from the scientific community in the areas of telecommunications and terrestrial broadcasting during the past few decades. Orthogonal Frequency Division Multiplexing (OFDM) is a key technology that is widely used in wireless communication systems [1]. However, one of the major drawbacks of the OFDM systems is the high peak-to-average power ratio (PAPR).

In comparison to the OFDM, recently it had been hypothesized that Orthogonal Time Frequency Space (OTFS) modulation is more resilient to channel-induced Doppler shift and is going to be the foundation for 6G [2]. This is one of the modulations having a two-dimensional (2D) domain and used by fixed wireless communications for the first time. The main concept involved in this modulation is the delay-Doppler domain. There are some works, where the OTFS is expected to be used in Radio Detection and Ranging (RADAR) [3]. OTFS guarantees the data rates with a high power gain. It is an extension of 2D Code Division Multiple Access (CDMA) [4] since it utilizes the basis functions that span the whole bandwidth and duration. However, unlike CDMA and OFDM, the set of OTFS basis functions is particularly designed to battle the dynamics of the time-varying multi-path channel. In essence, OTFS is built to inherit the good features of OFDM, Time Division Multiple Access (TDMA), and CDMA.

The one of the drawbacks of OTFS modulation is the PAPR [5]. In the literature, it has been mentioned that PAPR is small for the OTFS compared to OFDM [6]. It is known that a high amount of PAPR leads to less power efficiency. Hence

it requires to have less PAPR in the multi-carrier modulations including the OTFS scheme as well.

One of the techniques for reducing the PAPR in multi-carrier modulations is selected mapping (SLM) [7]. In this paper, we analyze the PAPR performance in OTFS modulation using classical SLM. This is proposed because the PAPR reduction can be done without any signal distortion throughout the process. This classical SLM technique uses the phase rotation process for the input and provides a good performance in the reduction of PAPR.

II. RELATED WORKS

The PAPR reduction techniques are classified into two types, Non-SLM and SLM techniques.

A. Non-SLM techniques

In [8], the authors proposed the PAPR reducing method called Compadding. The underlying process of this technique is to strengthen the weak amplitude signals and weaken the strong amplitude signals. This is done for altering the dynamic range of the signals. The PAPR has been reduced without affecting the Bit error rate (BER) by this companding technique. Block coding can also be used for PAPR reduction [9]. Here, the authors used the Arithmetic code and Reed-Solomon code for reducing PAPR by 18.5% and 16.7%. One more method for reducing the PAPR is using one of the matrices, i.e., the Hilbert matrix [10]. This is compared over the Circulant and Hadamard matrices, then it was declared that usage of the Hilbert matrix gives the best output in the reduction of PAPR.

B. SLM techniques

The SLM techniques are well known for PAPR reduction. In [11], it has shown that the SLM technique provides good performance in the PAPR reduction in OFDM systems. The other PAPR reduction technique used for the coded OFDM signal is the modified SLM [12]. This technique is not only used for the reduction of PAPR but also observed the improvement in the BER performance with a lossless data rate. One more technique in the same domain is novel improved SLM [13] which is used for the reduction of PAPR as well

as the best performance in the output BER at the receiver end. In search and partial implementation [14], this method is introduced for avoiding the usage of oversampling the signal for reducing the PAPR. This technique can be combined with the SLM technique for obtaining high PAPR performance with low complexity.

The main goal of this work is to reduce the PAPR in the OTFS modulated signals. To achieve this goal, we choose the classical SLM technique i.e., one of the types of selected mapping for reducing the PAPR. This classical SLM uses phase rotation of the input signal and then modulation is applied for that phase-rotated input signal. For that modulated signal, PAPR is calculated and finally, the least PAPR is selected and updated among all PAPR values and is noted as the final PAPR value.

III. SYSTEM MODEL

The Block representation of the OTFS is shown in Figure 1. In OTFS modulation, Quadrature Amplitude Modulation (QAM) symbols are used and by using the symbol mapper, time domain input is converted into a 2D delay-Doppler domain signal applied as input to the ISFFT block. The symbol mapper output is expressed as $x(k, l)$,

$$x(k, l) = \sqrt{T} \sum_{n=-\infty}^{\infty} x(k + nT) e^{-j2\pi n l T}, \quad (1)$$

$$T > 0, -\infty < k < \infty, -\infty < l < \infty$$

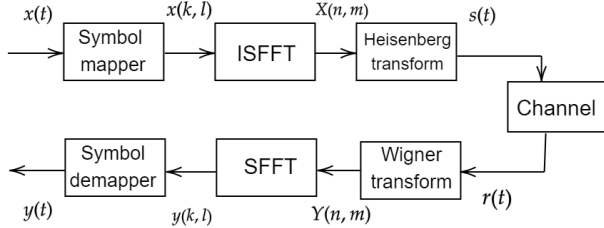


Figure 1 – Block representation of OTFS modulation and demodulation. [15]

Here in the OTFS modulation, both input i.e., $x[k, l]$ and output i.e., $y[k, l]$ as shown in the figure are in the delay-Doppler domain. As shown in Figure 1, in the flow of the OTFS block diagram, two types of Fast Fourier Transforms (FFT) are involved. They are Inverse Symplectic Finite Fourier Transform (ISFFT) and Symplectic Finite Fourier Transform (SFFT) [16]. These ISFFT and SFFT are used for mapping the symbols from the delay-Doppler domain i.e., $x[k, l]$ to the time-frequency domain i.e., $X[n, m]$ and from the time-frequency domain i.e., $Y(n, m)$ to the delay-Doppler domain i.e., $y(k, l)$ respectively. Figure 1 represents the whole process involved in the OTFS modulation. The input for this modulation is in the delay-Doppler domain then it will be converted into the time frequency domain. Then time-frequency signal is converted

into the time domain by applying the Heisenberg transform [16]. The output of the Heisenberg transform is the OTFS modulated signal. Then it is forwarded to the channel and again gets back to the time-frequency domain by using the Wigner transform [16]. Then, for obtaining the demodulated output in the delay Doppler domain SFFT is used. This is the complete process carried out for getting the OTFS modulated output signal. For a better understanding, the notations used in Figure 1 are explained in the following.

In $x(k, l)$ and $y(k, l)$, k, l represents the delay-Doppler domain. In $X(n, m)$ and $Y(n, m)$, (n, m) represents the time-frequency domain and in $s(t)$ and $r(t)$, t represents the time domain. x and X are on the modulated side and Y and y are at the demodulated side.

A. OTFS modulated signal

In this section, we discuss the OTFS modulated signal in the mathematical format and the input-output relations of the modulated signal. The modulation process is shown in Figure 1. As we mentioned earlier, OTFS is a 2D modulation, and the transmitter uses $k \times l$ grid of delay-Doppler for multiplexing of the information symbols (N, M) which are greater than zero. This (N, M) denoted by x belongs to the alphabet A i.e., QAM modulation alphabet which belongs to the points to the delay-Doppler grid in 2D ($k = 0, 1, \dots, N - 1$) and ($l = 0, 1, \dots, M - 1$). The time duration of the OTFS transmitted signal is given by $N \times T$ and transmitted signal Bandwidth (BW) = $M \Delta f$ [4]. Here N and M denote the number of input symbols and the length of sub carriers respectively, T is the symbol time and Δf is the frequency spacing. The below mathematical expressions are for the OTFS modulation and demodulation.

The conversion of delay-Doppler domain input signal $x(k, l)$ to time-frequency domain signal $X(n, m)$ is done by Inverse Symplectic Finite Fourier Transform (ISFFT) i.e.,

$$X(n, m) = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x(k, l) e^{j2\pi(\frac{nk}{N} - \frac{ml}{M})}. \quad (2)$$

Then the Heisenberg transform [16] is for converting the time frequency domain signal $X(n, m)$ to the time domain signal $s(t)$,

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X(n, m) g_{tx}(t - nT) e^{j2\pi m \Delta f (t - nT)}. \quad (3)$$

From the above equation, $s(t)$ is the OTFS modulated or transmitted signal. Here g_{tx} is the basic pulse of the transmission (for example, a rectangular pulse).

B. OTFS demodulated signal

The Wigner transform [16] is used for converting the time domain $r(t)$ to the time-frequency $Y(n, m)$,

$$Y(n, m) = A_{g_{rx}, r}(n, m) \triangleq \int g_{rx}^*(n' - n) r(n') e^{-j2\pi f(n' - n)} dt'. \quad (4)$$

The conversion of the time-frequency domain $Y(n, m)$ to the delay-Doppler domain $y(k, l)$ is done by Symplectic Finite Fourier Transform (SFFT) i.e.,

$$y(k, l) = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} Y(n, m) e^{-j2\pi(\frac{nk}{N} - \frac{ml}{M})}. \quad (5)$$

From the above equation, $y(k, l)$ is the OTFS demodulated signal. Finally, by using the symbol demapper, the delay Doppler domain is converted into the time domain and expressed as $y(t)$,

$$y(t) = \sqrt{T} \int_0^{\Delta f} y(k, l) dl, T > 0 \quad (6)$$

IV. PAPR REDUCTION USING CLASSICAL SELECTED MAPPING

Figure 2 shows the proposed approach of classical SLM applied for OTFS. The process initially starts with the phase rotation vector $\theta^{(u)}$ with the length of the sub carriers (M) expressed as,

$$\theta^{(u)} = e^{j\psi} \in \mathbb{C}^M, \quad u \in \{1, \dots, U\}, \quad (7)$$

where $\psi \in [0, 2\pi)$ and U is the SLM coefficient and it is represented in the powers of 2. Then rotate the input symbol vector using the phase rotation vector $\theta^{(u)}$.

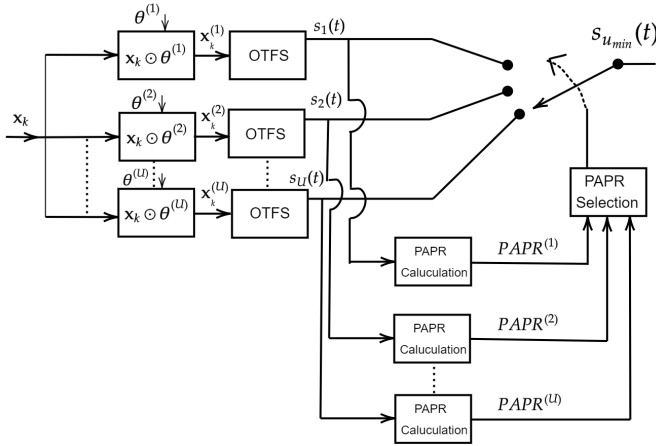


Figure 2 – Block representation of the classical selected mapping technique.

let \mathbf{x} be the input symbol vector for the k^{th} OTFS symbol and is expressed as

$$\mathbf{x}_k = (x_{k,0}, x_{k,1}, \dots, x_{k,l}, \dots, x_{k,M-1})^T, 0 \leq l \leq M-1. \quad (8)$$

The N complex input symbols with M sub carriers are expressed as the $x_{k,l}$ which is written as,

$$x_{k,l} = R_k^l + j.I_k^l, 0 \leq l \leq M-1, 0 \leq k \leq N-1, \quad (9)$$

where R_k^l and I_k^l are real and imaginary parts of the k^{th} symbol on the l^{th} sub carrier, respectively. From the above equation, Now, the phase rotated input symbol vector is expressed as,

$$\mathbf{x}_k^{(u)} = \mathbf{x}_k \odot \theta^{(u)}, \quad (10)$$

where \odot is the carrier wise point-to-point multiplication.

After the rotation of the phase vectors, OTFS modulation is performed. For that modulated signals, the PAPR is calculated by using the following equation [17] for each $u \in \{1, \dots, U\}$.

$$\text{PAPR}^{(u)} = \frac{\max_{0 \leq t \leq T} |s_u(t)|^2}{\frac{1}{T} \int_0^T |s_u(t)|^2 dt}, \quad (11)$$

where, $s_u(t)$ is the OTFS modulated signal, as per equation (2). For the OTFS modulation, the input is the rotated output from the equation (9) i.e., $\mathbf{x}_k^{(u)}$. After the calculation of PAPR, the final step is to select the least PAPR value among all the phase vectors ($\theta^{(u)}$). This is written as

$$\text{PAPR}_{u_{min}} = \min_{1 \leq u \leq U} \text{PAPR}^{(u)}, \quad u \in \{1, \dots, U\}. \quad (12)$$

Now, $s_{u_{min}}(t)$ is the OTFS modulated signal selected for transmission.

V. RESULTS

MATLAB simulations were performed with N number of symbols and different FFT sizes (M). However, we provide the results of 10000 symbols and 1024 FFT size. The results of the original OFDM and OTFS signals (i.e., without applying the SLM technique) are compared with the results of the SLM applied OFDM and OTFS signals. Figure 3 shows the simulation curves of the Complementary Cumulative Distribution Function (CCDF) of PAPR of OTFS and OFDM modulations. The CCDF of PAPR is a metric for analyzing PAPR, which is defined as the likelihood that the PAPR of a discrete temporal signal exceeds a certain threshold represented in decibels (dB). The plot is generated for the different SLM coefficient U values for both OFDM and OTFS modulations. In the CCDF plot, the solid line represents the OTFS signal and the dotted line represents the OFDM signal. The values of the PAPR (from the x-axis) are noted from the plot at the threshold of 10^{-3} (on the y-axis). These values are shown in the below-mentioned tables.

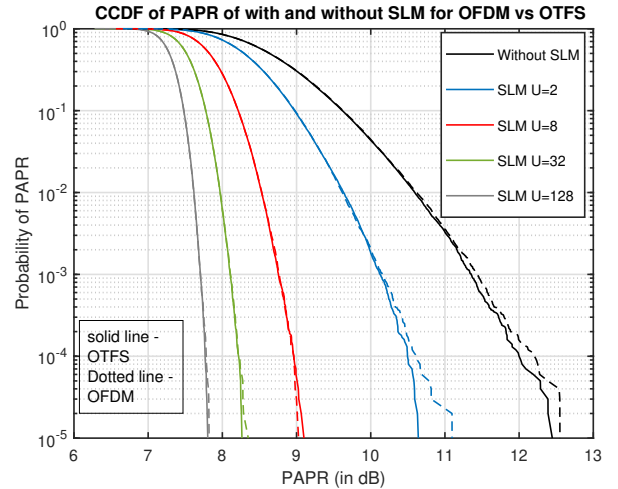


Figure 3 – CCDF Plot of PAPR for OFDM and OTFS.

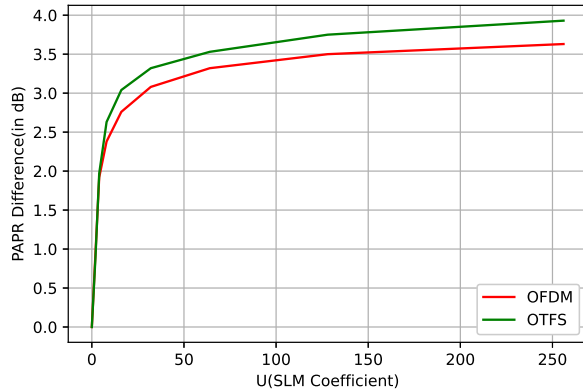


Figure 4 – PAPR difference with SLM coefficients.

Table I shows the numerical values of PAPR of both the modulations OFDM and OTFS with SLM (at $U = 2$ and 8) and without SLM which is considered as the original signal. Table II shows the reduction in PAPR in both OFDM and OTFS modulations by performing the subtraction of the PAPR values with the original values (i.e., values obtained without SLM) and the obtained values with SLM are tabulated in Table II. PAPR reduction values are calculated by using the original values and SLM technique values with the different U values are shown in Table II. At the 10^{-3} threshold, for $U=2$, the PAPR reduction of 1.2765 dB is achieved for OFDM. Whereas in OTFS, the PAPR reduction of 1.2131 dB is achieved. Further, for $U = 8$ PAPR reduction of 1.3877 dB and 1.3699 dB is achieved for OFDM and OTFS respectively. We have also calculated the PAPR difference for different SLM coefficients. Figure 4 shows the PAPR difference for different values of SLM coefficient U . It is seen that the PAPR difference is more while increasing the value of U . From this plot, we say that the PAPR performance is better in the OTFS when compared to OFDM.

TABLE I – PAPR values with and without SLM in OFDM and OTFS modulations

Modulation	PAPR at CCDF= 10^{-3} (in dB)		
	Without SLM	With SLM, $U=2$	With SLM, $U=8$
OFDM	11.4309	10.1544	8.7666
OTFS	11.3390	10.1259	8.7559

TABLE II – PAPR reduction with SLM in OFDM and OTFS modulations

Modulation	PAPR Reduction (in dB)	
	SLM, $U=2$	SLM, $U=8$
OFDM	1.2765	1.3878
OTFS	1.2131	1.3700

VI. CONCLUSION

In this paper, the classical selected mapping method to diminish the PAPR in OTFS modulation is introduced. The

results are compared for both the OFDM and OTFS modulations. From the obtained results, we observe that by applying the classical SLM the PAPR performance is better when compared to the results obtained without applying classical SLM for both OFDM and OTFS. We observed that increasing the SLM coefficient results in better performance in terms of PAPR reduction for the OTFS modulated signals. Hence, we conclude that the classical SLM technique can be a promising technique for PAPR reduction in OTFS modulation.

REFERENCES

- [1] Hernán-Felipe Arraño-Scharager and Cesar Azurdia-Meza. OFDM: Today and in the future of next generation wireless communications. In *35th IEEE Convencion de Estudiantes de Centroamerica y Panama (CONESCAPAN)*, 09 2016.
- [2] Suvra Sekhar Das and Ramjee Prasad. *OTFS: Orthogonal Time Frequency Space Modulation A Waveform for 6G*, pages i–xxvi. 2021.
- [3] P. Raviteja, Khoa T. Phan, Yi Hong, and Emanuele Viterbo. Orthogonal time frequency space (OTFS) modulation based radar system. In *2019 IEEE Radar Conference (RadarConf)*, pages 1–6, 2019.
- [4] Ronny Hadani, Shlomo Rakib, Michail Tsatsanis, Anton Monk, Andrea J Goldsmith, Andreas F Molisch, and R Calderbank. Orthogonal time frequency space modulation. In *2017 IEEE Wireless Communications and Networking Conference (WCNC)*, pages 1–6. IEEE, 2017.
- [5] Mengxue Liu, Ming-Min Zhao, Ming Lei, and Min-Jian Zhao. Autoencoder based PAPR reduction for OTFS modulation. In *2021 IEEE 94th Vehicular Technology Conference (VTC2021-Fall)*, pages 1–5, 2021.
- [6] GD Surabhi, Rose Mary Augustine, and Ananthanarayanan Chockalingam. Peak-to-average power ratio of OTFS modulation. *IEEE Communications Letters*, 23(6):999–1002, 2019.
- [7] SS Krishna Chaitanya Bulusu, Hmaied Shaiek, Daniel Roviras, and Rafik Zayani. Reduction of papr for FBMC-OQAM systems using dispersive SLM technique. In *2014 11th international symposium on wireless communications systems (ISWCS)*, pages 568–572. IEEE, 2014.
- [8] Cheemala Naveen and V Sudha. Peak-to-average power ratio reduction in OTFS modulation using companding technique. In *2020 5th International Conference on Devices, Circuits and Systems (ICDCS)*, pages 140–143. IEEE, 2020.
- [9] Azlina Idris, Nur Lisa Mohd Sapari, Mohd Syarhan Idris, Suzie Seroja Sarmin, Wan Norsyafizan Wan Mohamad, and Nani Fadzlina Naim. Reduction of PAPR using block coding method and APSK modulation techniques for F-OFDM in 5G system. In *TENCON 2018 IEEE Region 10 Conference*, pages 2456–2460. IEEE, 2018.
- [10] M Palanivelan, Dr Sheila Anand, and M Gunasekaran. Matrix based low complexity PAPR reduction in OFDM systems. *International Journal of Electronics & Communication Technology*, 2(2):158–162, 2011.
- [11] Kavita Mhatre and Uday Pandit Khot. Efficient selective mapping PAPR reduction technique. *Procedia computer science*, 45:620–627, 2015.
- [12] Si-Yu Zhang and Behnam Shahrava. A SLM scheme for PAPR reduction in polar coded OFDM-IM systems without using side information. *IEEE Transactions on Broadcasting*, 67(2):463–472, 2020.
- [13] Jian-guo Yuan, Qian Shen, Jing-xin Wang, Yong Wang, Jin-zhao Lin, and Yu Pang. A novel improved SLM scheme of the PAPR reduction technology in CO-OFDM systems. *Optoelectronics Letters*, 13(2):138–142, 2017.
- [14] Chin-Liang Wang, Sheng-Ju Ku, and Chun-Ju Yang. A low-complexity PAPR estimation scheme for OFDM signals and its application to SLM-based PAPR reduction. *IEEE Journal of Selected Topics in Signal Processing*, 4(3):637–645, 2010.
- [15] MK Ramachandran, GD Surabhi, and A Chockalingam. OTFS: A new modulation scheme for high-mobility use cases. *Journal of the Indian institute of science*, 100(2):315–336, 2020.
- [16] KR Murali and A Chockalingam. On OTFS modulation for high-doppler fading channels. In *2018 Information Theory and Applications Workshop (ITA)*, pages 1–10. IEEE, 2018.
- [17] Hideki Ochiai and Hideki Imai. On the distribution of the peak-to-average power ratio in OFDM signals. *IEEE transactions on communications*, 49(2):282–289, 2001.