Efficient Iterative Massive MIMO Detection Using Chebyshev Acceleration

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***Abstract*—Massive multiple-input multiple-output (MIMO) detection is one of the most important, yet complex parts of the fifth generation (5G) baseband receiver. The linear minimum mean square error (MMSE) signal detection achieves almost optimum efficiency when the number of antennas at the base station is asymptotically large. However, the matrix inversion required for MMSE can also be very complex when the number of users increases. In this paper, a low complexity signal detection algorithm based on modified accelerated overrelaxation (MAOR) method is proposed to iteratively approach the MMSE performance. We calculate optimal values of two key parameters of MAOR and also provide a suitable and less complex initial solution to accelerate the convergence. Furthermore, we adopt the Chebyshev polynomial acceleration technique to present the MAOR method with a new vector combinations, which enhances the performance of the detection algorithm. The spectral radius of MAOR is also calculated to demonstrate its suitability for Chebyshev acceleration. This complete solution is referred to as Chebyshev-MAOR. The results have revealed that the proposed method can achieve faster convergence and better performance than other state-of-the-art detection algorithms. It is also shown that Chebyshev-MAOR reduces computational complexity by an order of magnitude from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$, with K denoting the number of transmit antennas. Our performance results show that these complexity gains are achieved with negligible impact on the bit error rate (BER) performance.**

***Index Terms*—Massive-MIMO, approximate matrix inversion, matrix decomposition, QR, LDL, Cholesky.**

I. Introduction

High data rate and reliability are essential requirements for fifth generation (5G) and future wireless networks due to the increasing number of connected devices. Massive multiple-input multiple-output (MIMO) is a promising technology to fulfill these requirements for next-generation of communication systems. In massive MIMO, the base station (BS) is equipped with hundreds of antennas to serve tens or hundreds of users in the same time-frequency resources, as well as providing substantial improvements in link reliability, spectral efficiency, energy efficiency and interference reduction [1], [2]. Massive MIMO scales up the capacity and performance advantages of traditional small-scale MIMO systems, where the typical number of antennas is relatively small. Despite all the advantages, massive MIMO suffers from a high computational complexity associated with a large number of antennas. A major challenge for baseband processing of a massive MIMO originates from the symbol vector detection in the uplink (UL) receiver [3] due to the substantially increased channel dimensions and multiuser interferences. For this reason, to reduce the BSs' complexity and cost, it is required to relax further the computational complexities related to linear schemes. The optimum detection algorithm in multi-user scenarios is the maximum likelihood (ML) detector. However, it suffers from exponential complexity in terms of the number of users (and the constellation size), which make the ML detector impractical for a large number of antennas [4]. Some nonlinear detectors such as fixed-complexity sphere decoding (SD) [5], tabu search [6], and the belief propagation (BP) algorithms [7] can be found in the literature. Although these solutions can approach the optimum performance ML, the computational complexity is still extremely high, precluding its use when the channel dimensions is very high, as in the massive MIMO case, especially for high modulation orders. Therefore, linear detection methods, such as zero-forcing (ZF) and minimum mean square error (MMSE) are the natural choice due to the good performance/complexity tradeoffs. However, for massive MIMO systems even linear detectors can be too complex because they require the inversion of large matrices. For example, a 16-user massive MIMO system requires a 16×16 matrix inversion. It should be noted that linear detectors are near-optimal for certain massive MIMO configurations, namely when the ratio of numbers between (BS) antennas and users is high. Due to channel hardening, the uncorrelated noise and the small-scale fading can be eliminated. As a result, a linear signal detector such ZF and MMSE can achieve near-optimal performance. The linear detectors are easily implementable for a small-scale MIMO system but suffers from high complexity in a massive MIMO setting due to the high computational complexity associated with the matrix inversion when the number of users is high. Therefore, a new class of detectors based on approximate matrix inversion have become popular over the past decade. For example, a Neumann series approximation (NSA) scheme was proposed in [8] to convert the matrix inverse operation into a series of matrix-vector multiplications by utilizing the diagonal matrix. The successive overrexlation (SOR) has been proposed in [9] to balance the tradeoff of performance and complexity by an optimal relaxation parameter to achieve better performance and avoid the inversion matrix in an iterative way, but still need larger

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iterations to achieve the some performance. The Gauss-Seidel (GS) method basd on detection signal proposed in [10] with a proper initial solution to attain the target performance with low computational complexity, but the internal parts of each (GS) iteration render parallel execution complicated, and hence, the computational complexity is raised. The error recovery-based detector [11], Richardson iteration [12] and conjugate gradient [13] are also iterative detection methods, which utilize the initial solution to achieve near-MMSE performance without a matrix inversion, but they suffer from signal-to-noise ratio (SNR) loss. Interested readers are invited to read through [14] to know more about the approximate inversion based detectors. The message passing-based detection (MPD) [15] used to simplify the calculation of matrix-inverse in massive MIMO system, however it contains many exponential computations caused by increasing the number of users and the order of modulation. In [16] and [17], the Chebyshev iteration and Newton iteration (NI) have been proposed to provide fast convergence while their complexities depend on the number of iterations. Furthermore, these iterative methods require a complex calculation of initial input to ensure convergence. However, designing efficient methods for accelerating the iterative method, which are needed to improve the convergence and reduce the complexity, can be challenging [18], [19]. The Chebyshev acceleration [19] uses a polynomial, involving a new sequence vector from linear combinations obtained from the basic iterative method, to achieve faster convergence. Therefore, Chebyshev acceleration based on successive over-relaxation (SOR) and symmetric successive over-relaxation (SSOR) approaches have been proposed to achieve near optimal detection for massive MIMO systems [18], [20]. In [21], a modified accelerated overrelaxation (MAOR) method is proposed for iterative solution of nonsingular linear systems, which is a generalization of accelerated overrelaxation (AOR) [22], [23]. (MAOR) essentially develops a new splitting matrix which depends on M-splittings of a singular matrix [24], [25]. The convergence of the (MAOR) method can be controlled by the selection of its acceleration and relaxation parameters. Therefore, the splitting matrix based on (MAOR) is able to present a polynomial acceleration such as Chebyshev. Also, the small spectral radius of (MAOR) makes the convergence rate better than conventional AOR and (SOR) when using acceleration and relaxation parameters [21]. In contrast to the studies above, in this paper, we propose a low-complexity massive MIMO signal detection algorithm based on (MAOR) method [21]. The (MAOR) based iterative detection is improved further by deriving optimal values for two key parameters of the original (MAOR) algorithm, i.e., the acceleration and relaxation parameters. In addition, an eigenvalue based initial solution is applied instead of utilizing the diagonal matrix, to achieve a better convergence rate for the MAOR. A derivation of the spectral radius of the iteration matrix of (MAOR) is also provided, which is used for polynomial acceleration. Finally, the Chebyshev polynomial acceleration method is applied to (MAOR) iterations to reach near-MMSE performance. It is shown that Chebyshev acceleration has a relatively simple structure with the aid of error analysis. The resulting Chebyshev accelerated MAOR, which we refer to as Chebyshev-MAOR, provides near MMSE performance with reduced complexity by an order of magnitude. For the sake of comparison, the performance of the Chebyshev-MAOR is compared with state-of-the-art massive MIMO detectors for similar channel conditions, iteration numbers, and MIMO configurations. The contributions of this work are as follows.

- A novel low-complexity efficient data detection algorithm is proposed based on the modified accelerated overrelaxation (MAOR) method iteration, which enables near-MMSE error rate performance for uplink M-MIMO systems.
- We present the optimal values of two key parameters of (MAOR) and develop a suitable and less complex initial solution to make the proposed algorithm more appealing for realistic implementations. This has shown that it achieves the fastest convergence of existing counterparts.
- We demonstrate that by using the Chebyshev acceleration technique, the proposed algorithm overcomes the susceptibility of the traditional (MAOR) detector to relaxation parameters while significantly accelerating the convergence rate.
- The computational complexity of aforementioned various approaches are examined and numerical models are used to explain efficiency disparities.

The results show that the Chebyshev-MAOR has better performance because of the superior convergence properties. To our best knowledge, this is the first attempt to apply Chebyshev acceleration and MAOR method jointly for a massive MIMO detection algorithm. The remainder of the paper is organized as follows. Section II describes the system model which defines UL and channel models of massive MIMO and also the linear detector method. Section III presents the application of (MAOR) for massive MIMO detection, explains how to select the acceleration and relaxation parameter and an appropriate initial solution. Section IV presents the polynomial acceleration method using Chebyshev and complexity analysis of the proposed method. Section VI presents error-rate performance results of the Chebyshev-MAOR method and compares them with state-of-the-art solutions . Finally, conclusions are drawn in Section VII.

Throughout this paper we employ the following notations. Lower-case boldface and upper-case boldface letters denote vectors and matrices, respectively. $(.)^T, (.)^H, (.)^{-1}$ refer to the transpose, the matrix Hermitian transpose and matrix inversion, respectively. $\mathcal{O}(.)$ stands for the order of magnitude. $\rho(.)$ denotes the spectral radius of a matrix.

II. System Model

We consider the UL model of massive MIMO as shown in Fig. 1. The BS is equipped with N number of antennas to simultaneously serve K users equipment (UE), where N is much larger than K, $N \gg K$ [26]. Each user k uses the same time-frequency resources, which means that the users transmit data to the BS simultaneously and at the same frequencies. The bit stream transmitter and encoder channel is mapped to the constellation M-QAM. The transmitted signal $\mathbf{x} \in \mathbb{C}^{K\times 1}$ can be expressed as $\mathbf{x} = [x_1, x_2, ..., x_K]^T$, and the received signal

$\mathbf{y} \in \mathbb{C}^{N\times 1}$ can be presented as $\mathbf{y} = [y_1, y_2, ..., y_N]^T$. Hence, The received signal at the BS can be given as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N\times K}$ indicates the UL channel matrix, $\mathbf{H} = [h_1, h_2, ..., h_N]^T$, with $\mathbf{h}_N \in \mathbb{C}^{K\times 1}$ denoting the channel vector between the user and BS, $\mathbf{n}$ is the additive white Gaussian noise (AWGN) with distribution $\mathcal{CN}\ (0, \sigma^2)$.

Fig. 1: The considered UL massive MIMO system.

A. Massive MIMO Channel Model

In this work, we consider Rayleigh fading channel and a spatially correlated channel, which are commonly employed in massive MIMO [27]. The correlated channel is presented as a Kronecker channel model [28], which incorporates spatial correlation between antenna elements. It is also assumed that the spatial correlations at the transmitter and receiver are separable. The Kronecker channel model $\check{\mathbf{H}}$ can be illustrated as

$$\mathbf{H} = \sqrt{\mathbf{R}_{Rx}}\check{\mathbf{H}}\sqrt{\mathbf{R}_{Tx}}, \tag{2}$$

where $\mathbf{R}_{Rx} \in \mathbb{C}^{K\times K}$ and $\mathbf{R}_{Tx} \in \mathbb{C}^{N\times N}$ denote the transmit correlation matrix and the receive correlation matrix and $\check{\mathbf{H}} \in \mathbb{C}^{N\times K}$ is a matrix with independent and identically distributed (i.i.d.) complex Gaussian random variables (RVs) with zero mean and unit variance (i.i.d.) $\mathcal{CN}\ (0, 1)$. We assume exponential correlation matrices [28], which means that the (p, q) element of $\mathbf{R}_{Tx}$ is given by

$$\mathbf{R}_{Tx}(q, p) = (\zeta_{Tx})^{q-p} \tag{3}$$

where ζ_{Tx} indicates a correlation parameter between two adjacent antennas ($0 \leq |\zeta_{Tx}| \leq 1$) and its phase can take any value in [0,2π]). For $|\zeta_{Tx}| = 0$ there is no correlation between transmit antennas. Similarly, the (p, q) term of the receive correlation matrix is

$$\mathbf{R}_{Rx}(q, p) = (\zeta_{Rx})^{q-p} \tag{4}$$

with the correlation parameter ζ_{Rx}.

B. Linear Detector

The optimal ML detection minimizes the squared Euclidean distance between the actual received vector $\mathbf{y}$ and the hypothesized received signal $\mathbf{Hx}$, the ML solution is given by

$$\mathbf{x}^{ML} = \underset{\mathbf{x}\in\mathbb{C}^K}{\arg\min} ||\mathbf{y} - \mathbf{Hx}||_2^2. \tag{5}$$

However, the ML receiver has complexity which grows exponentially with the number of users and the constellation size, which precludes its use even in MIMO scenarios with a moderate number of users. MMSE equalization based detection is widely used due to to its good performance/complexity tradeoffs. The MMSE estimate of the transmitted signal vector $\hat{x}_{MMSE}$ is

$$\begin{aligned}\hat{\mathbf{x}}_{MMSE} &= (\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I_N})^{-1}\mathbf{H}^H\mathbf{y} \\ &= (\mathbf{G} + \sigma_n^2\mathbf{I_N})^{-1}\mathbf{H}^H\mathbf{y} = \mathbf{W^{-1}}\mathbf{y}^{MF},\end{aligned} \tag{6}$$

where $\mathbf{y}^{MF} = \mathbf{H}^H\mathbf{y}$ is the output of matched filter, $\mathbf{W}$ is the MMSE filter matrix and $\mathbf{G} = \mathbf{H}^H\mathbf{H}$ is the Gram matrix of the channel [29], [30]. From (6), it is clear that the MMSE receiver involves the inversion of a matrix whose dimensions grow with the number of users, which might be too complex for massive MIMO systems. The MMSE detection in (6) requires an inversion of $\mathbf{W}$ which has $\mathcal{O}(K^3)$ complexity and leads to inefficient and power hungry realization in a massive MIMO system. The column vectors of the channel matrix are asymptotically orthogonal [3], but even for smaller dimensions, a Gram matrix is always Hermitian and (at least) positive semi-definite (i.e., the eigenvalues are greater or equal to 0). This means that $\mathbf{G} + \sigma_n^2\mathbf{I_N}$ is positive definite, since $\sigma_n^2 > 0$.

III. Low-Complexity Signal Detection for UL Massive MIMO

In this section, we present the MAOR based massive MIMO detection algorithm. To accelerate the convergence rate and reduce the computational complexity, suitable values of acceleration and relaxation parameter are identified. Spectral radius of Jacobi matrix is exploited to achieve a high convergence rate. In addition, a method for selecting the initial solution is proposed. Finally, the Chebyshev polynomial acceleration is utilized to further improve the convergence rate.

A. Itearative Detection based on the MAOR

The estimated signal in (6) can be obtained by the MAOR because of the symmetric positive definite (SPD) property. It is well known that the MAOR method is widely used to solve the N-dimension linear equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is the $N \times N$ symmetric positive definite matrix, $\mathbf{x}$ is the $N \times 1$ solution vector, and $\mathbf{b}$ is the $N \times 1$ measurement vector. It can efficiently solve the linear system without exact computation of matrix inversion. Since the matrix $\mathbf{W}$ is SPD, it can be decomposed to a diagonal component $\mathbf{D}$, a strictly lower triangular component $\mathbf{L}$, and a strictly upper triangular $\mathbf{U}$. Then, $\mathbf{W}$ can be decomposed as:

$$\mathbf{W} = \mathbf{D} - \mathbf{L} - \mathbf{U}, \tag{7}$$

The MAOR is based on M-splittings nonsingular [25] as follows

$$\mathbf{W} = \mathbf{M} - \mathbf{N}, \tag{8}$$

where the singular M-matrix and N-Matrix are presented as

$$\mathbf{M} = \frac{1}{\alpha}(\mathbf{D} - \beta\mathbf{L}), \mathbf{N} = \frac{1}{\alpha}((1-\alpha)\mathbf{D} + (\alpha - \beta)\mathbf{L} + \alpha\mathbf{U}). \tag{9}$$

Then, the MAOR iterative method [21] can be presented as

$$\mathbf{x}_{i+1} = \mathbf{M}^{-1}\mathbf{N}\mathbf{x}_i + \mathbf{M}^{-1}\mathbf{y}^{MF}. \tag{10}$$

Consequently,

$$\begin{aligned}\mathbf{x}_{i+1} = (\frac{1}{\alpha}(\mathbf{D} - \beta\mathbf{L}))^{-1}(\frac{1}{\alpha}((1-\alpha)\mathbf{D} \\ +(\alpha - \beta)\mathbf{L} + \alpha\mathbf{U})\mathbf{x}_i) + (\frac{1}{\alpha}(\mathbf{D} - \beta L))^{-1}\mathbf{y}^{MF},\end{aligned} \tag{11}$$

where i, α, β and $\mathbf{x}_0$ are the number of iterations, acceleration parameter, relaxation parameter and initial solution, respectively [21]. For $\alpha = \beta$, we have a modified SOR (MSOR) converge for the some value of α [31]. The modified Gauss-Seidel (MGS) method ($\alpha = \beta = 1$) converges and the Modified Jacobi (MJ) method ($\beta = 0, \alpha = 1$) also converges [31]. The iteration matrix of MAOR, $\mathbf{M}^{-1}\mathbf{N}$ can be expressed as

$$\mathbf{B}_{MAOR} = (\frac{1}{\alpha}(\mathbf{D} - \beta\mathbf{L}))^{-1}(\frac{1}{\alpha}((1-\alpha)\mathbf{D} + (\alpha - \beta)\mathbf{L} + \alpha\mathbf{U})). \tag{12}$$

If the spectral radius of MAOR achieves $\rho(\mathbf{B}_{MAOR}) < 1$ or $\lim_{k\to\infty} \mathbf{B}^k_{MAOR} = 0$, the iterative method will converge for each kth user. According to random matrix theory [32], the spectral radius of $\mathbf{B}_{MAOR}$ is given by

$$\rho(\mathbf{B}_{MAOR}) = \max |\lambda(\mathbf{B}_{MAOR})|, \tag{13}$$

where $\lambda(\mathbf{B}_{MAOR})$ donates the eigenvalue of matrix $\mathbf{B}_{MAOR}$. It is clear that the MAOR performance and complexity profile depends on α and β. They can be selected based on the spectral radius of Jacobi iteration matrix, $\rho(\mathbf{B}_{Jacobi})$ [21] to accelerate convergence rate. The estimation of the optimum acceleration and relaxation parameters is done in the following subsection.

B. Optimum Parameters

From (11), it is clear that the selection of the acceleration and relaxation parameters will influence the speed of convergence rate of the proposed MAOR technique. The eigenvalue λ of MAOR iteration matrix is defined according to [33] as

$$(\lambda + \beta - 1)^2 = (\alpha\lambda + \beta - \alpha)\beta\rho^2(\mathbf{B}_{Jacobi}). \tag{14}$$

According to (13), to achieve a fast convergence rate of proposed method, we need to minimize of the eigenvalue λ, to get a smallest spectral radius. Therefore, the optimal parameters of the proposed method are [21], [34]

$$\beta^{opt} = \frac{2}{1 + \sqrt{1 - \rho^2(\mathbf{B}_{Jacobi})}}, \tag{15}$$

$$\alpha^{opt} = \frac{\rho^2(\mathbf{B}_{Jacobi}) - \rho^4(\mathbf{B}_{Jacobi})}{\rho^2(\mathbf{B}_{Jacobi})(1 - \rho^2(\mathbf{B}_{Jacobi}))}, \tag{16}$$

where $\rho(\mathbf{B}_{Jacobi})$ indicates the spectral radius of Jacobi iteration matrix. To find the optimal acceleration α^{opt} and relaxation β^{opt} parameters we need to obtain the spectral radius of Jacobi method. The Jacobi iterative method can be expressed as

$$\mathbf{x}_{(i+1)} = (\mathbf{I} - \mathbf{D}^{-1}\mathbf{W})\mathbf{x}_i + \mathbf{D}^{-1}\mathbf{y}^{MF}, \tag{17}$$

where $\mathbf{D}$ is diagonal matrix of $\mathbf{W}$ and $\mathbf{I}$ is identity matrix. The iteration matrix of Jacobi is defined as

$$\mathbf{B}_{(Jacobi)} = \mathbf{I} - \mathbf{D}^{-1}\mathbf{W}, \tag{18}$$

and the corresponding spectral radius of MAOR is $\rho(\mathbf{B}_{MAOR}) = \beta^{opt} - 1$ [35]. Since computing the eigenvalues of matrix $\mathbf{W}$ is complicated, an approximation is adopted here. As the number of antennas N and users K increase then the approximate eigenvalues of the matrix can be calculated as

$$\rho(\mathbf{B}_{Jacobi}) = \rho(-\mathbf{D}^{-1}\mathbf{W} - \mathbf{I}_K) = \rho(-\mathbf{D}^{-1}\mathbf{W}) - 1. \tag{19}$$

Since $N >> K$, the elements of diagonal matrix $\mathbf{D}$ will convergence to N. Thus, for uplink massive MIMO the channel is asymptotically orthogonal when $N >> K$. Thus, we can conclude that $\mathbf{W}^{-1}$ is diagonally dominant. This special property inspires us to utilize $\mathbf{W}^{-1} = \mathbf{D}^{-1} = \frac{1}{N}\mathbf{I}_K$, and (19) becomes

$$\rho(\mathbf{B}_{Jacobi}) = \frac{1}{N}\rho(-\mathbf{W}) - 1. \tag{20}$$

Based on the random matrix theory [32]. The spectral radius of $\mathbf{W}$ and the largest and smallest eigenvalues of $\mathbf{W}$ can be approximated as

$$\rho(-\mathbf{W}) = |\lambda_{max}(\mathbf{W})| = N\left(1 + \sqrt{\xi}\right)^2, \tag{21}$$

$$\rho(-\mathbf{W}) = |\lambda_{min}(\mathbf{W})| = N\left(1 - \sqrt{\xi}\right)^2, \tag{22}$$

where $\xi = N/K$ is the ratio between the number of antennas and the number of users. Finally, by substituting (21) and (22) into (20), the spectral radius of the iteration matrix $\rho(\mathbf{B}_{Jacobi})$ can be approximated as

$$\rho(\mathbf{B}_{Jacobi}) = \overline{\eta} = \left(1 + \sqrt{\xi}\right)^2 - 1, \tag{23}$$

$$\rho(\mathbf{B}_{Jacobi}) = \underline{\eta} = \left(1 - \sqrt{\xi}\right)^2 - 1, \tag{24}$$

Since the number of antennas N is much larger than the number of users K, which makes the value of ξ almost constant, and indicates that with the increase of the loading factor $\xi = N/K$, the spectral radius of $\rho(\mathbf{B}_{Jacobi})$ decreases [35]. Thus, a faster convergence rate can be achieved. However, as the spectral radius is related the value of the matrix's eigenvalues, α and β can be rewriten as

$$\alpha^{opt} = \frac{2}{1 + \sqrt{1 - \overline{\eta}^2}}, \tag{25}$$

$$\beta^{opt} = \frac{\overline{\eta}^2 - \underline{\eta}^4}{\overline{\eta}^2\left(1 - \underline{\eta}^2\right)}. \tag{26}$$

However, the condition

$$0 \le \alpha \le \beta, \tag{27}$$

should be satisfied. It should be pointed out that the careful selection of the optimum parameters is critical to a high performance and low complexity of the detection.

C. Approximate Initial Solution

Even with the the optimal parameters α and β, the proposed method still needs an iterative initial solution which provides a faster and desired result. Theoretically, any moderate initial solution can lead to the convergence of iterative method, although it might require a substantially higher number of iterations. Since the channel matrix $\mathbf{H}$ is asymptotically orthogonal, for $N > K$ the condition of favorable channel can be described as

$$\frac{\mathbf{h}_j^H \mathbf{h}_k}{N} \to 0, j \neq k, j, k = 1, 2, ...K, \tag{28}$$

where $\mathbf{h}_j$ represents the jth column of $\mathbf{H}$. The inverse of matrix $\mathbf{W}$ is diagonally dominant, so when the ratio of N/K increases, the non-diagonal matrix $\mathbf{W^{-1}}$ and the diagonal one $\mathbf{D}^{-1}$ become increasingly similar, so that matrix $\mathbf{W}$ is a diagonally dominant matrix and satisfies [3] as

$$\mathbf{W}_{(j,k)} = \begin{cases} \frac{\lambda_{max}+\lambda_{min}}{2} & i = k, \\ 0, & i \neq k. \end{cases} \tag{29}$$

Therefore, we propose the initial solution proposed as

$$\mathbf{x}_0 = \frac{2}{\lambda_{max} + \lambda_{min}} \mathbf{H}^H \mathbf{y}. \tag{30}$$

This initial value enables the proposed iterative method to achieve a faster convergence rate. The computational complexity of the initial value calculation is significantly small if we follow (20) and (21). In addition, the computation of the initial value can be executed in parallel, which is an important advantage form the hardware implementation point of view.

D. Convergence Analysis

The proposed method achieves a sufficient condition of convergence rate when the spectral radius of the iteration matrix is less than 1. Additionally, the approximation error is important element to determine the convergence rate. Then the approximation error of the proposed MAOR can be expressed as

$$\hat{\mathbf{x}}^{(n+1)} - \hat{\mathbf{x}} = \mathbf{B}_{\alpha,\beta}(\hat{\mathbf{x}}^{(n)} - \hat{\mathbf{x}}) = ... = \mathbf{B}_{\alpha,\beta}^{(n+1)}(\hat{\mathbf{x}}^{(0)} - \hat{\mathbf{x}}). \tag{31}$$

The approximation error can be expressed as

$$\begin{aligned} \left\| \hat{\mathbf{x}}^{(n+1)} - \hat{\mathbf{x}} \right\|_2 &= \left\| \mathbf{B}_{\alpha,\beta}^{(\mathbf{n+1})} \right\|_F \left\| \hat{\mathbf{x}}^{(0)} - \hat{\mathbf{x}} \right\|_2 \\ &\le \|\mathbf{B}_{\alpha,\beta}\|_F^{(n+1)} \left\| \hat{\mathbf{x}}^{(0)} - \hat{\mathbf{x}} \right\|_2. \end{aligned} \tag{32}$$

The determination of the approximation error of the proposed MAOR detection scheme is affected by the Frobenius norm of the iteration matrix $\mathbf{B}_{\alpha,\beta}$. A smaller $||\mathbf{B}_{\alpha,\beta}||_F$ provides a faster convergence for the proposed method. In massive MIMO, the number of antennas at the BS N grows to infinity with a fixed number of users K and $\beta = 1$, which means that the proposed method MAOR can have faster convergence rate than Neumann series (NS) which define $2 - \sqrt{2} \le \alpha \ge \sqrt{2}$. Hence, we need to show that that the Frobenius norms of the iteration matrices of the NS method are larger than that of the proposed MAOR method. It is clear that $||\mathbf{B}_{\alpha,\beta}||_F \ge ||\mathbf{B_{NS}}||_F$, which $||\mathbf{B_{NS}}||_F$ represents the iteration matrix of the NS $\mathbf{B_{NS}} = \mathbf{D^{-1}}(\mathbf{L} + \mathbf{U})$, then the Frobenius norm is [36]

$$\begin{aligned} ||\mathbf{B}_{NS}||_F = ||\mathbf{D}^{-1}(\mathbf{L} + \mathbf{U})|| &= \left[\sum_{m=1}^{K} \sum_{m=1}^{K} \left| \frac{\mathbf{w}_{m,m}}{\mathbf{w}_{m,m}} \right|^2 \right]^{\frac{1}{2}} \\ &= \sqrt{2}||\mathbf{D}^{-1}L||_F, \end{aligned} \tag{33}$$

where $\mathbf{w}_{m,m}$ denotes the mth column of matrix $\mathbf{W}$ and

$$||\mathbf{D}^{-1}L||_F = ||\mathbf{D}^{-1}\mathbf{U}||_F.$$

From the iterative matrix $\mathbf{B}_{\alpha,\beta}$ when $\beta = 1$, we have

$$\begin{aligned} \mathbf{B}_{MAOR} &= (\frac{1}{\alpha}(\mathbf{D} - \mathbf{L}))^{-1}(\frac{1}{\alpha}((1-\alpha)D + (\alpha-1)\mathbf{L} + \alpha\mathbf{U})) \\ &= (\frac{\mathbf{D}^{-1}}{\alpha}(\mathbf{I}_K - \mathbf{D}^{-1}\mathbf{L}))^{-1} \\ &\times (\frac{1}{\alpha}((1-\alpha)\mathbf{D} + \alpha\mathbf{L} - \mathbf{L} + \alpha\mathbf{U})) \\ &= (\frac{1}{\alpha}(\mathbf{I}_K - \mathbf{D}^{-1}L))^{-1} \\ &\times \mathbf{D}^{-1}(\frac{1}{\alpha}((1-\alpha)\mathbf{D} + \alpha\mathbf{L} - \mathbf{L} + \alpha\mathbf{U})) \\ &= (\frac{1}{\alpha}(\mathbf{I}_K - \mathbf{D}^{-1}\mathbf{L}))^{-1} \\ &\times (\frac{1}{\alpha}((1-\alpha) + \mathbf{D}^{-1}\alpha\mathbf{L} - \mathbf{D}^{-1}L + \mathbf{D}^{-1}\alpha\mathbf{U})). \end{aligned} \tag{34}$$

The matrix $(\mathbf{I}_K - D^{-1}\mathbf{L}))^{-1}$ can be calculated by the following polynomial expansion

$$\begin{aligned} (\mathbf{I}_K - \mathbf{D}^{-1}\mathbf{L}))^{-1} &= \sum_{k=0}^{\infty}(-1)^k(\mathbf{D}^{-1}\mathbf{L})^k \\ &= \mathbf{I}_K + \mathbf{D}^{-1}\mathbf{L} + \sum_{k=2}^{\infty}(-1)^k(\mathbf{D}^{-1}\mathbf{L})^k. \end{aligned} \tag{35}$$

Assuming a massive MIMO where the number of BS antennas much larger than UEs K i.e, $N \gg K$, the upper valued L goes to zero and diagonal part goes to N, which defined as $\lim_{k\to\infty} \mathbf{D}^{-1}\mathbf{L} = 0$. Therefore, (34) can be approximated as

$$\begin{aligned} \mathbf{B}_{MAOR} &\approx (\frac{1}{\alpha}(\mathbf{I}_K + \mathbf{D}^{-1}\mathbf{L})) \\ &\times (\frac{1}{\alpha}((1-\alpha) + \mathbf{D}^{-1}\alpha\mathbf{L} - \mathbf{D}^{-1}\mathbf{L} + \mathbf{D}^{-1}\alpha\mathbf{U})) \\ &\approx (\frac{1}{\alpha}((\frac{1}{\alpha}((1-\alpha) + \mathbf{D}^{-1}\alpha\mathbf{L} - \mathbf{D}^{-1}\mathbf{L} + \mathbf{D}^{-1}\alpha\mathbf{U})). \end{aligned} \tag{36}$$

By using Frobenius norm theory to approximation $\mathbf{B}_{MAOR}$ can be selected as

$$||\mathbf{B}_{\alpha,\beta}||_F = ||(\frac{1}{\alpha}((\frac{1}{\alpha}((1-\alpha)+\alpha\mathbf{L} - \mathbf{D}^{-1}\mathbf{L}+\mathbf{D}^{-1}\alpha\mathbf{D}^{-1}\mathbf{U}))||_F = \frac{1}{\alpha^2}\bigg(1-\alpha+\alpha||\mathbf{D}^{-1}\mathbf{L}||_F - ||\mathbf{D}^{-1}L||_F+\alpha||\mathbf{D}^{-1}L||_F\bigg) = \frac{1}{\alpha^2}(1-\alpha)+\frac{1}{\alpha^2}(\alpha-1+\alpha)||\mathbf{D}^{-1}\mathbf{L}||_F = \sqrt{2}(\frac{1}{\alpha^2}(1-\alpha)+\frac{1}{\alpha^2}(\alpha-1+\alpha))/2||\mathbf{D}^{-1}\mathbf{L}||_F = \sqrt{2}(\frac{1}{\alpha^2}(1-\alpha)+\frac{1}{\alpha^2}(\alpha-1+\alpha))/2 \leq 1 \Rightarrow 2-\sqrt{2}\leq\alpha\geq\sqrt{2}. \quad (37)$$

Then, we conclude that $||\mathbf{B}_{\alpha,\beta}||_F \leq ||\mathbf{B}_{NS}||_F$ which shows that the Frobenius norms of the iterative matrix of the proposed method has smallest the spectral radius compared to the iterative method of NS method. To demonstrate that $||\mathbf{B}_{\alpha,\beta}||_F \leq ||\mathbf{B}_{MSOR}||_F$, where $||\mathbf{B}_{MSOR}||_F = (\mathbf{D}-\beta\mathbf{L}^{-1}) \times (\mathbf{L}^H+(1-\beta)\mathbf{L})$, we can prove the following based on the spectral radius of MSOR:

$$||\mathbf{B}_{MSOR}||_F = (\mathbf{D}-\beta(\mathbf{L}^{-1}(\mathbf{L}^H+(1-\beta)\mathbf{L}) = \mathbf{D}^{-1}(\mathbf{I}-\beta(\mathbf{L}^{-1})(\mathbf{U}+(1-\beta)\mathbf{L}) = (\mathbf{I}-\beta(\mathbf{L}^{-1})(\mathbf{D}^{-1}\mathbf{L}^H+(1-\beta)\mathbf{D}^{-1}\mathbf{L}). \quad (38)$$

From (35), the spectral radius of MSOR can be simplified as

$$\approx \mathbf{D}^{-1}\mathbf{L}^H+(1-\beta)\mathbf{D}^{-1}\mathbf{L}) \approx ||1-\beta||\mathbf{D}^{-1}\mathbf{L}^H+\mathbf{D}^{-1}\mathbf{L} \approx (||1-\beta||+1)\mathbf{D}^{-1}\mathbf{L} \approx (||1-\beta||+1)||\mathbf{R}_{NS}||_F. \quad (39)$$

In [18], the value of β in MSOR is limited to $1<\beta<2$, where MAOR has $0<\alpha<\beta$, which makes the contain of $(||1-\beta||+1)||$ large. Thus, $||\mathbf{B}_{\alpha,\beta}||_F \leq ||\mathbf{B}_{MSOR}||_F$, as equation (39) contains NS which is large spectral radius then MAOR. It has been demonstrated that MAOR method converges faster than MSOR and can even converges in situations when when the MSOR failes to converge [21]. To further speed up the convergence rate, we will apply the Chebyshev polynomial acceleration method described below.

IV. Chebyshev Acceleration

The convergence rate of our method can be further improved by adopting an polynomial acceleration scheme. Chebyshev is well known acceleration method, which has been rigorously discussed in [19] and [37], [38]. Suppose, we convert $\mathbf{x}_{MMSE}\mathbf{W} = \mathbf{y}^{MF}$ to the iteration following as [19]

$$\mathbf{x}_{i+1} = \mathbf{B}_{MAOR}\mathbf{x}_i+\mathbf{c}, c = (\frac{1}{\alpha}(\mathbf{D}-\beta\mathbf{L}))^{-1}\mathbf{y}^{MF}, \quad (40)$$

where $\mathbf{B}_{MAOR} = \mathbf{M}^{-1}\mathbf{N}$ and $\mathbf{c} = \mathbf{N}^{-1}\mathbf{y}^{MF}$. Then, we use (40) to give all these approximations of signal vector $\mathbf{x}$. Consequently, the Chebyshev acceleration is good method to accelerate the convergence $(\mathbf{x}_i)_{i=0}^{\infty}$. Based on the signal vector sequence $(\mathbf{x}_i)_{i=0}^{\infty}$, we construct a new linear combination $(\mathbf{y}_m)_{i=1}^{\infty}$, which features faster convergence towards the accurate solution $\mathbf{x}_i$ of (40) and is known as secondary iteration, as

$$\mathbf{y}_m = \sum_{i=1}^{m}\gamma_{m,i}\mathbf{x}_i, \quad (41)$$

where parameters $\gamma_{m,i}$ can be referenced in polynomials as,

$$\mathbf{p}_m(t) = \sum_{i=1}^{m}\gamma_{m,i}\mathbf{B}^i. \quad (42)$$

Note that the scalars $\gamma_{m,i}$ must satisfy $\sum_{i=0}^{m}\gamma_{m,i}=1$. Since at convergence $\mathbf{x_0}=\mathbf{x_1}=...=\mathbf{x}$ have been generated via the iteration the equation $\mathbf{p}_m=\mathbf{x}$ should be applied. Then the error approximations of $\mathbf{x}$ and $\mathbf{y}_m$ provide as

$$\mathbf{y}_m-\mathbf{x} = \sum_{i=1}^{m}\gamma_{m,n}(\mathbf{x}_i-\mathbf{x}) = \sum_{i=1}^{m}\gamma_{m,i}\mathbf{B}^i_{MAOR}(\mathbf{x}_0-\mathbf{x}) = \mathbf{p}_m(\mathbf{B}_{MAOR})(\mathbf{x}_0-\mathbf{x}), \quad (43)$$

where $\mathbf{p}_m(\mathbf{B}_{MAOR}) = \sum_{i=1}^{m}\gamma_{m,i}\mathbf{B}^i_{MAOR}$ is polynomial of degree m with $p_m(1)=1$, and $\mathbf{p}_m(\mathbf{z}) = \sum_{i=1}^{m}\gamma_{m,i}\mathbf{z}^i$
The mth expression of Chebyshev polynomials are defined by the three-term recurrence relation as

$$\mathbf{T}_{m+1}(\mathbf{z}) = 2\mathbf{z}T_m(\mathbf{z})-\mathbf{T}_{m-1}(\mathbf{z}), m\geq 1, \quad (44)$$

where initial parameters indicates to $\mathbf{T_0}(\mathbf{x})=\mathbf{1}$ and $\mathbf{T}_1(z)=z$. The Chebyshev polynomials have many interesting properties. A polynomial parameter indicated as

$$\mathbf{p}_m(z) = \frac{\mathbf{T}_m(z/\rho)}{\mathbf{T}_m(1/\rho)}, \quad (45)$$

can provide minimum error, while ρ is spectral radius of matrix $\mathbf{B}_{MAOR}$ belonging to $[\lambda_{min},\lambda_{max}]$. The optimal iteration polynomials $\mathbf{p}_m(\mathbf{z})$ can be obtained by the Chebyshev polynomials as

$$\mathbf{p}_m(z) = \frac{\mathbf{T}_m(1+2\frac{z-\epsilon}{\epsilon-\eta})}{\mathbf{T}_m(1+2\frac{r-\epsilon}{\epsilon-\eta})}, \quad (46)$$

where ϵ and η are satisfied as

$$\epsilon = \frac{\lambda_{max}+\lambda_{min}}{\lambda_{max}-\lambda_{min}}, \quad \eta = \frac{\lambda_{max}+\lambda_{min}}{2}. \quad (47)$$

The ϵ and η donate the smallest and the largest eigenvalues of the iteration matrix $\mathbf{M}$, respectively. Here, r is any scalar satisfying the coefficients of polynomials. Let us assume that $\mu_m = 1/\mathbf{T}_m(1/\rho)$, so $\mathbf{p}_m(\mathbf{B}_{MAOR}) = \mu_m/T_m(\mathbf{B}_{MAOR})/\rho)$, and $\gamma_{0,0}=1, \gamma_{1,0}=0, \gamma_{1,1}=1$. Then, the properties of μ_m becomes

$$\frac{1}{\mu_{m+1}} = 2\frac{1}{\rho\mu_m}-\frac{1}{\mu_{m-1}}, \quad (48)$$

To express the mth Chebyshev polynomial method to purposed method, we rewrite the equation of y_m, which also satisfies a three-term-recurrence as

$$\begin{aligned}\mathbf{y}_{m+1}-\mathbf{x} &= \mathbf{p}_m(\mathbf{B}_{MAOR})(\mathbf{x}_0-\mathbf{x}) = \mu_m\mathbf{T}_m(\frac{\mathbf{B}_{MAOR}}{\rho})(\mathbf{x_0}-\mathbf{x})\\ &= \mu_m\Big[2.\frac{\mathbf{B}_{MAOR}}{\rho}.\mathbf{T}_m(\frac{\mathbf{B}_{MAOR}}{\rho})(\mathbf{x_0}-\mathbf{x})\\ &\quad -\mathbf{T}_{m-1}(\frac{\mathbf{B}_{MAOR}}{\rho})(\mathbf{x_0}-\mathbf{x})\Big]\\ &= \mu_m\Big[2.\frac{\mathbf{B}_{MAOR}}{\rho}.\frac{\mathbf{p}_m(\frac{\mathbf{B}_{MAOR}}{\rho})(\mathbf{x_0}-\mathbf{x})}{\mu_m}\\ &\quad -\frac{\mathbf{p}_{m-1}(\frac{\mathbf{B}_{MAOR}}{\rho}))(\mathbf{x_0}-\mathbf{x})}{\mu_{m-1}}\Big]\\ &= \mu_m\Big[2.\frac{\mathbf{B}_{MAOR}}{\rho}.\frac{\mathbf{y}_m-\mathbf{x}}{\mu_m}-\frac{\mathbf{y}_{m-1}-\mathbf{x}}{\mu_{m-1}}\Big]\\ &= 2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{B}_{MAOR}\mathbf{y}_m-\frac{\mu_{m-1}}{\mu_{m+1}}\mathbf{y}_{m-1}+\mathbf{d}_m. \end{aligned} \quad (49)$$

From (48) , the rest of properties can be defined as

$$\begin{aligned}\mathbf{d}_m &= \mathbf{x}-2\frac{\mu_m}{\mu_{m+1}}\frac{\mathbf{B}_{MAOR}}{\rho}\mathbf{x}-\frac{\mu_{m-1}}{\mu_{m+1}}\mathbf{x}\\ &= \mathbf{x}-2\frac{\mu_m}{\mu_{m+1}}\frac{\mathbf{x}-\mathbf{c}}{\rho}\mathbf{x}-\frac{\mu_{m-1}}{\mu_{m+1}}\mathbf{x}\\ &= \mu_m\Big(\frac{1}{\mu_{m+1}}-\frac{2}{\rho\mu_m+\frac{1}{\mu_{m-1}}}\Big)\mathbf{x}+2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{c}\\ &= 2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{c}. \end{aligned} \quad (50)$$

After some mathematical manipulations, the secondary iteration can be defined by

$$\mathbf{y}_{m+1} = 2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{B}_{MAOR}\mathbf{y}_m-\frac{\mu_{m-1}}{\mu_{m+1}}y_{m-1}+2\frac{\mu_m}{\rho\mu m+1}\mathbf{c}, \quad (51)$$

where $y_0 = x_0$ and $y_1 = x_1$. The Chebyshev polynomial can be easily handled to MAOR method, which based on spectral radius ρ and Iteration matrix $\mathbf{B}_{MAOR}$.
We present the complete Chebyshev accelerated MAOR, which we refer to as Chebyshev-MAOR, in Algorithm 1.

Algorithm 1 The Chebyshev-MAOR

Input: $\mathbf{y}, \mathbf{H}, \mathbf{W}, \sigma^2, n, \beta, \alpha$
Output: Estimated signal $\mathbf{y}_{Iteration}$
Primary iteration:
$\mathbf{x}_{i+1} = (\frac{1}{\alpha}(\mathbf{D}-\beta\mathbf{L}))^{-1}(\frac{1}{\alpha}((1-\alpha)\mathbf{D}+(\alpha-\beta)\mathbf{L}+\alpha\mathbf{U})\mathbf{x}_i)+(\frac{1}{\alpha}(\mathbf{D}-\beta\mathbf{L}))^{-1}\mathbf{y}^{MF}$.
Initialization:
$\mathbf{x}_0 = \mathbf{y}_0 = \frac{2}{\lambda_{max}+\lambda_{min}}\mathbf{H}^H\mathbf{y}^{MF}$.
$\mathbf{y}_1 = \mathbf{B}_{MAOR}\mathbf{X}_i+\mathbf{c}$.
$\mu_0 = 1; \mu_1 = \rho$.
$0 \le \alpha \le \beta$.
Iteration:
for i = 1 : m
$\frac{1}{\mu_{m+1}} = 2\frac{1}{\rho\mu_m}-\frac{1}{\mu_{m-1}}$;
$\mathbf{y}_{m+1} = 2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{B}_{MAOR}\mathbf{y}_m-\frac{\mu_{m-1}}{\mu_{m+1}}y_{m-1}+2\frac{\mu_m}{\rho\mu m+1}\mathbf{c}$.
end
Return $\mathbf{y}_{Iteration}$.

A. Error Analysis

The error formula is critical to show the merits of the Chebyshev acceleration method [39]. Assume to the matrix $\mathbf{B} = \mathbf{M^{-1}N}$ is symmetric and its eigenvalues satisfy

$$-1 < \lambda_{min} < ... < \lambda_{max} < 1. \quad (52)$$

Let us consider the error formula defined as

$$\mathbf{e}_i = \mathbf{x_0}-\mathbf{x}, \mathbf{e}_{\mathbf{y}_i} = \mathbf{y}_i-\mathbf{x}, i = 0, 1, 2, ... \quad (53)$$

The eigenpair of matrix $\mathbf{B}$ donate as (λ_i, v_i) which v is an orthonormal basis of real value matrix of $\mathbf{B}$ since is symmetric. Then $\mathbf{e_0}$ can be expressed as

$$\mathbf{e}_0 = \sum_{i=1}^{m} d_i v_i. \quad (54)$$

Let $\mathbf{p}_m$ be a polynomial with degree m and (λ_i, v_i) are eigenpair of matrix $\mathbf{B}$ and d_i scalar factor. Then

$$||\mathbf{e_y}^r||^2 = \sum_{i=1}^{m}[d_i^2 p_m^2(\lambda_i)], r = 1, 2, 3, ..., \quad (55)$$

noting that

$$\mathbf{x}_i-\mathbf{x} = (\mathbf{M}^{-1}\mathbf{N})^i(\mathbf{x}_0-\mathbf{x}) = \mathbf{B}^i(\mathbf{x}_0-\mathbf{x}). \quad (56)$$

Therefore, we have

$$\mathbf{y}_r-\mathbf{x} = \sum_{j=1}^{m}\gamma_j(\mathbf{x}_j-\mathbf{x}) = \sum_{j=1}^{m}\gamma_j\mathbf{B}^j(\mathbf{x}_0-\mathbf{x}) = \mathbf{p}_m(\mathbf{B})(\mathbf{x}_0-\mathbf{x}). \quad (57)$$

Thus,

$$||\mathbf{y}_r-\mathbf{x}||_2 \le ||\mathbf{p}_m(\mathbf{B})||_2||\mathbf{x}_0-\mathbf{x}||_2. \quad (58)$$

To simplify (58), we rewrite the equation as

$$\mathbf{ey}_r = \mathbf{p}_m(\mathbf{B})\mathbf{e}_0, ||\mathbf{ey}_0|| \le ||\mathbf{p}_m(\mathbf{B})||_2||\mathbf{e}_0||. \quad (59)$$

The error estimation in (59) is not very accurate. Hence, the structure needs to exploit a relaxed version of (54) and (57). The error can be express as

$$\begin{aligned}\mathbf{e}_{\mathbf{y}_r} &= p_m(\mathbf{B})\mathbf{e}_0 = \mathbf{p}_m(\mathbf{B})\sum_{i=1}^{m}d_i v_i\\ &= \sum_i d_i\mathbf{p}_m(\mathbf{B})v_i = \sum_i d_i(\sum_j\gamma_j\Lambda_j^j)v_i, \end{aligned} \quad (60)$$

which leads to

$$||\mathbf{e}_0||^2 = \sum_{i=1}^{m}d_i^2, ||\mathbf{e}_{\mathbf{y}_r}||^2 = \sum_{j=1}^{m}d_i^2\mathbf{p}_m^2(\lambda_i). \quad (61)$$

The improved approximation $\mathbf{y}_i$ then performs the same iteration and refinement to get another improved approximation $\mathbf{y}_{r+1}$,

$$||\mathbf{e}_{\mathbf{y}_{r+1}}||^2 = \sum_{i=1}^{m}[d_i^2\mathbf{p}_m^4(\lambda_i)], \quad (62)$$

where d_i (i=1,2,3,...) are the original values. The above process is repeated r times. Then the error estimation can be indicated as

$$||\mathbf{e_{y_r}}||^{\mathbf{2}} = \sum_{\mathbf{i=1}}^{\mathbf{m}}[\mathbf{d_i^2p_m^{2r}}(\lambda_\mathbf{i})], r = 1, 2, 3, ... \quad (63)$$

V. Computational Complexity

In this section we provide insights on the computational complexity of proposed method Chebyshev-MAOR. We express the complexity in terms of complex-valued multiplications which is a fairly standard practice in the literature. For the initial solution, which is based on a diagonally dominant of the filtering matrix MMSE $\mathbf{W}^{-1}$, it requires $K+1$ operations. The overall complexity of proposed method can be analysed from

$$\mathbf{y}_m^{i+1} = \frac{1}{\mathbf{w}_{m,m}}\Bigg[\beta\sum_{k=1}^{m+1}\mathbf{w}_{m,k}y_k^{i+1} + 2\frac{\mu_m}{\rho\mu_{m+1}}(1-\alpha)\mathbf{w}_{m,m}\mathbf{y}_m^i + 2\frac{\mu_m}{\rho\mu_{m+1}}(\alpha-\beta)\sum_{k=1}^{m+1}\mathbf{w}_{m,k}y_k^i + 2\frac{\mu_m}{\rho\mu_{m+1}}\alpha\sum_{k=m+1}^{K}\mathbf{w}_{m,k}\mathbf{y}_k^i - \frac{\mu_{m-1}}{\mu_{m+1}}\alpha\mathbf{w}_{m,m}\mathbf{y}_m^{i-1} + \beta\alpha\sum_{k=1}^{m+1}\mathbf{w}_{m,k}\mathbf{y}_k^{i-1} + 2\frac{\mu_m}{\rho\mu_{m+1}}\mathbf{y}^{MF}\Bigg]. \tag{64}$$

The required number of multiplications can be calculated as following: The term outside the square braces is 1 and the terms inside the braces require m, 2, $m+1$, $K-m-1$, 2, m and 1 operations, respectively. Hence, $\mathbf{y}_m^{i+1}$ requires $K+2m+6$ multiplications per iteration. The total complexity of the Chebyshev-MAOR is $2K^2+7K$ per iteration since m takes values from 1 to K. Hence, the analysis shows that the complexity order of the proposed method is $\mathcal{O}(K^2)$, which is lower than the conventional MMSE procedure, which has an $\mathcal{O}(K^3)$ complexity. Although other iterative methods can also reduce the complexity to $\mathcal{O}(K^2)$, but they still require a large number of iterations, which compromises the overall complexity. For example, the modified successive overrexlation (MSOR) and the modified accelerated overrexlation (MAOR) methods satisfy the complexity order which reduce $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$. The NS complexity conducted $\mathcal{O}(K^3)$, which more than two iterations (i.e., $n>2$). The proposed method provides a significant reduction in the signal processing complexity due to the smaller required number of iterations. Table 1 shows the overall complexities of the proposed method in addition to other algorithms. The order magnitude of MAOR, MSOR and NS have the some complexity $\mathcal{O}(K^2)$. However, the Neumann series based algorithms has $\mathcal{O}(K^2)$ only for $i=2$, while the order magnitude growths to $\mathcal{O}(K^3)$ when $i\geq 2$.

TABLE I: Computational complexity comparison

Method	Number of multiplications
MSOR [18]	$i\frac{3}{2}K^2 + i\frac{3}{2}K$
MAOR	$\frac{i}{2}(7k^2+3K)$
Chebyshev-MAOR	$i(2K^2+7K)$
Neumann Series [8]	$(i-2)K^3+NK^2+KN$
Chebyshev-MSOR [18]	$(16K+8K^2)i$
MMSE	iK^3+iK^2

VI. Results and Discussions

In this section, the error-rate performance of Chebyshev-MAOR and compare with other state-of-the-art detection

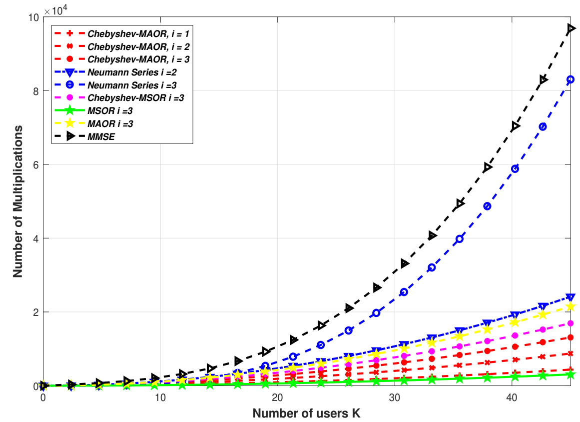


Fig. 2: The computational complexity versus the number of users.

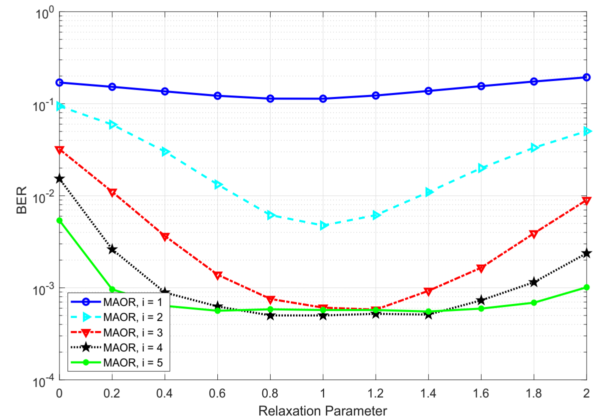
techniques are presented. To be more specific, the proposed Chebyshev-MAOR is compared with different variants of MSOR, Chebyshev-MSOR based detectors [18], Neumann series approximation [8], and exact inversion based conventional MMSE detection. Considering to massive MIMO scenarios corresponding to the UL transmission with K single antenna users and a BS with N antennas. Moreover, in the modulation 64-QAM and 256-QAM scheme is employed and also assume perfect synchronization of Monte Carlo simulations. The correlated channel model of (2) is employed, where the Rayleigh fading is considered $\zeta_{Tx}=\zeta_{Rx}=0$. As detection for UL massive MIMO, the BS-Correlated channel is consider to $\zeta_{Rx}=0.4$ and $\zeta_{Rx}=0.6$. The optimal relaxation parameter is β^{opt}= 1.14 and the optimum acceleration parameter is α^{opt} = 1.09, which can be computed using (25) and (26).

Figs. 3 and 4 shows the impact of the relaxation and acceleration range on the BER performance of the iterative method MAOR, for a SNR level of 15 dB with the Rayleigh fading channel and the Kronecker channel . Although there is some tolerance around the optimum value, the BER performance can be significantly affected by a poor choice of these parameters. We consider $N=128$ and $K=16$, for a 64-QAM constellation, although similar conclusions could be drawn for other constellations and antenna configurations. Hence, the value of relaxation plays important role for converges rate for the proposed method MAOR, while the acceleration value also effective the convergence rate but it is much less sensitive compare with relaxation parameters. Moreover, the impact of the relaxation and acceleration parameters are more degradation BER since apply the Kronecker channel, which obvious the Kronecker channel has a deeply degradation then the Rayleigh fading channel.

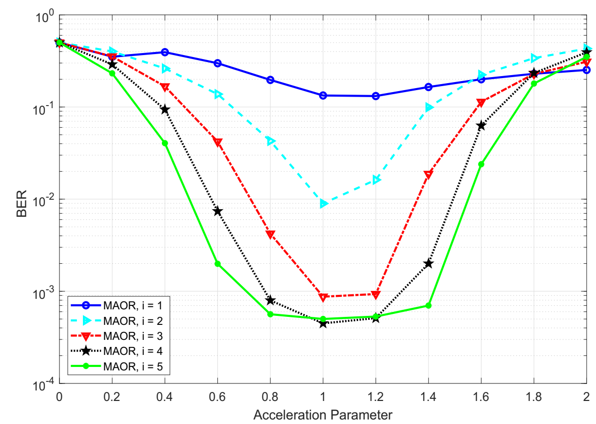
Figs. 5 and 6 compare the BER performance of proposed Chebyshev-MAOR algorithm for 128×16 antenna configuration (i.e., $\frac{N}{K}=\frac{128}{16}=8$) with different constellation size (64-QAM, 256-QAM). It is evident from the simulations that the proposed method provides substantially better performance than other methods. In fact, the proposed method achieves the ideal linear MMSE linear performance with just 2 iterations for 64-QAM, while the Chebyshev-MSOR method and other existing methods require a much larger number of iterations. Even when we increase the constellation size to 256-QAM the performance of the proposed method is still very close to the optimum MMSE performance, without requiring more iterations, contrarily to the other techniques, where the performance degradation can increase substantially. Additionally, Neumann series approximation is worst method to achieve the performance.

Fig. 7 presents the BER performance for 64×16 antenna configuration (i.e., $\frac{N}{K} = \frac{64}{16} = 4$) for a 64-QAM scheme.It is obvious that as the number of iterations increases, the BER efficiency of all methods approaches that of the MMSE perfermance. However, the Chebyshev-MAOR still significantly outperforms the other methods. as Fig. 7, when i=3, the SNR required by the Chebyshev-MAOR method to achieve the BER of 10^{-3} is 18 dB, whereas for the Chebyshev-MSOR method required SNR 19 dB. The proposed method achieved a quasi-MMSE performance in i=3 iterations.

Figs. 8 and 9 provide BER comparison of the proposed method and state-of-the-art detection techniques for $\frac{N}{K} = \frac{256}{32} = 8$ and $\frac{256}{64} = 4$ antenna configurations. Once again, Chebyshev-MAOR achieves a quasi-MMSE performance, although we need more iterations when $\frac{N}{K}$ is smaller. In addition, although the performance of the Neumann-based algorithm improves as i increases, it still faces a negligible performance loss. Next, we consider the impact of antenna correlations. Fig. 10 shows the impact of channel correlation on the performance of the proposed Chebyshev-MAOR when $\zeta_{Rx} = 0.4$ and $\zeta_{Rx} = 0.6$. We consider $\frac{N}{K} = \frac{128}{16} = 8$ for the ratio of numbers of BS antennas and number of users. The modulation scheme is 64-QAM. As expected, the MMSE algorithm degrades when the channel correlation increases, but the proposed method can still converge to the MMSE performance. Contrary to other scenarios, the required number of iterations increases slightly, although the complexity of the proposed method is still much lower than the other methods.



(a) $\alpha = 1.09$



(b) $\beta = 1.14$

Fig. 3: Efficient relaxation and acceleration range for the MAOR iterative method with configuration antenna N=128, K=16, 64 QAM, the Rayleigh fading channel adopted and with difference number of iterations i and SNR = 15dB: (a) impact of β for $\alpha = 1.09$; (b) impact of α for $\beta = 1.14$.

VII. Conclusions

In this paper, we proposed an efficient iterative massive MIMO detection algorithm based MAOR method. Speed of convergence for the proposed MAOR-based iterative detection is controlled by two key parameters (known as acceleration and relaxation parameters), and their optimum values were derived. In addition, we applied an eigenvalue-based initial solution instead of the common diagonal matrix approximation, which led to improved convergence rate. For the acceleration of the MAOR algorithm, we applied Chebyshev polynomial acceleration method that allows us to to reach near-MMSE performance with MAOR. We showed that Chebyshev acceleration has a relatively simple structure with the aid of error analysis. Our performance results showed that the resulting Chebyshev accelerated MAOR provides near MMSE performance with reduced complexity, outperforming state-of-the-art massive MIMO detectors for similar channel conditions.

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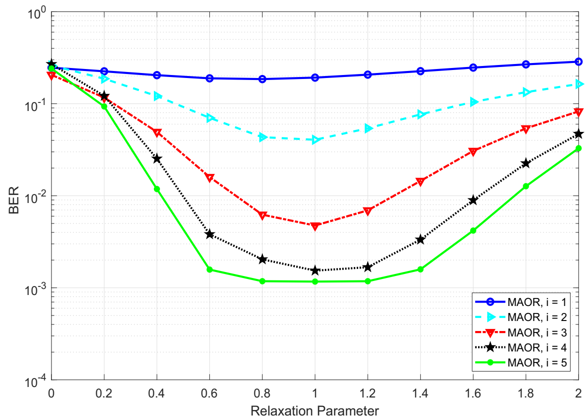
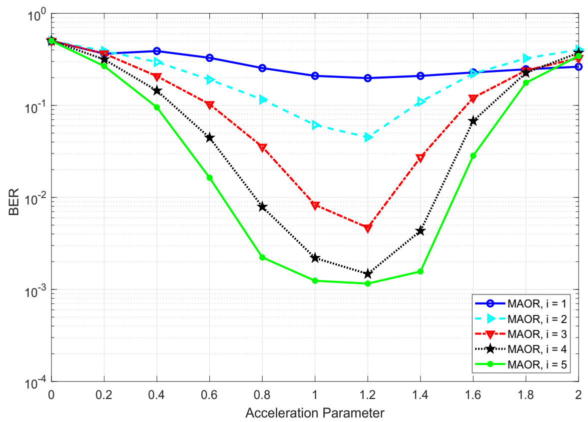


(a) $\alpha = 1.09$



(b) $\beta = 1.14$

Fig. 4: Efficient relaxation and acceleration range for the MAOR iterative method with configuration antenna N=128,K=16, 64 QAM, The Kronecker channel adopt with BS-Correlated $\zeta_{Rx} = 0.4$ and with difference number of iterations i and SNR = 15dB: (a) impact of β for $\alpha = 1.09$; (b) impact of α for $\beta = 1.14$.

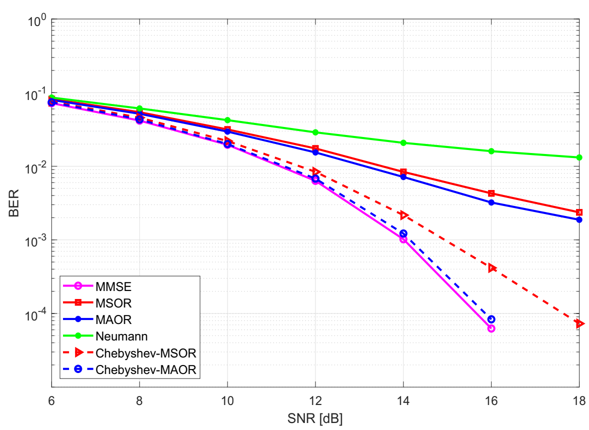


Fig. 5: BER performance for 64-QAM in a $N \times K = 128 \times 16$ scheme and i=2 iterations.

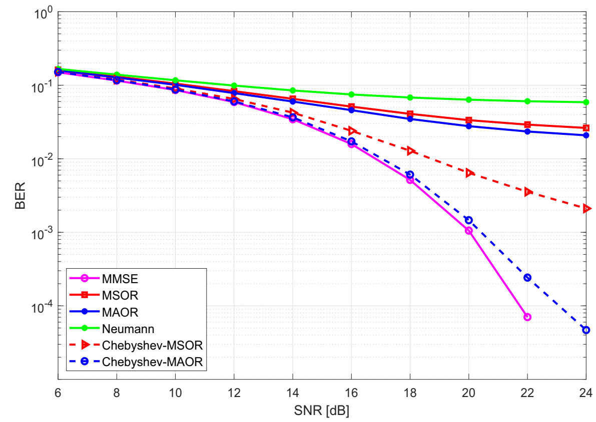


Fig. 6: BER performance for 256-QAM in a $N \times K = 128 \times 16$ scheme and i=2 iterations.

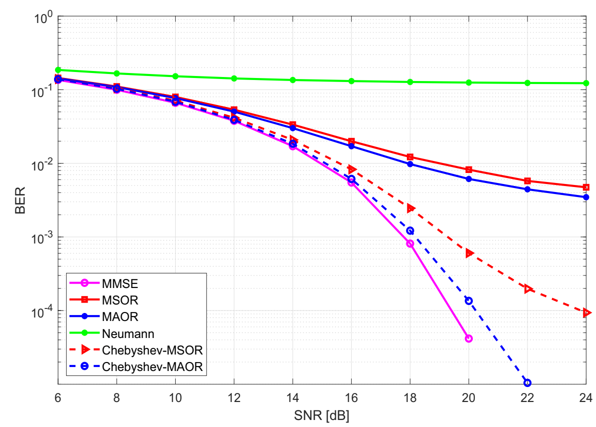


Fig. 7: BER performance for 64-QAM in a $N \times K = 64 \times 16$ scheme and i=3 iterations.

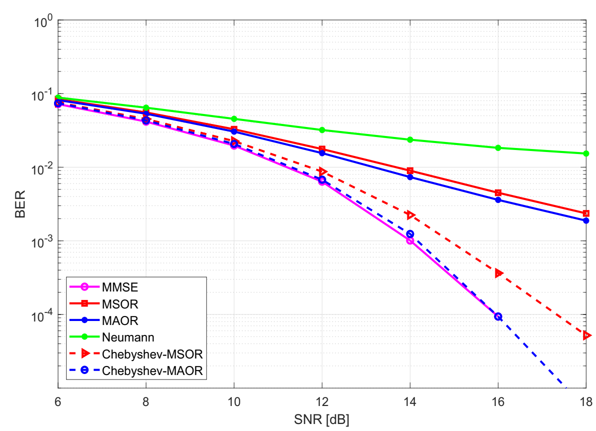


Fig. 8: BER performance for 64-QAM in a $N \times K = 256 \times 32$ scheme and i=2 iterations.

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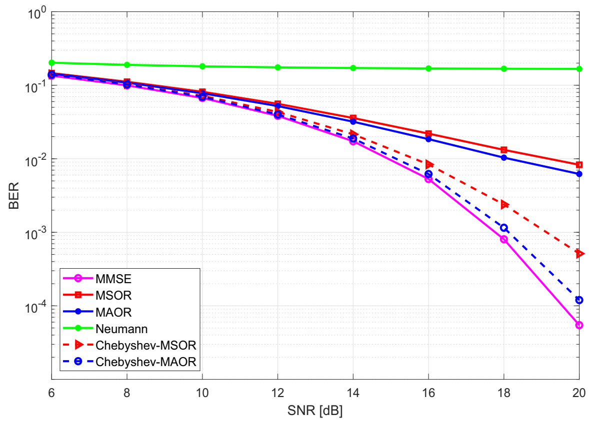
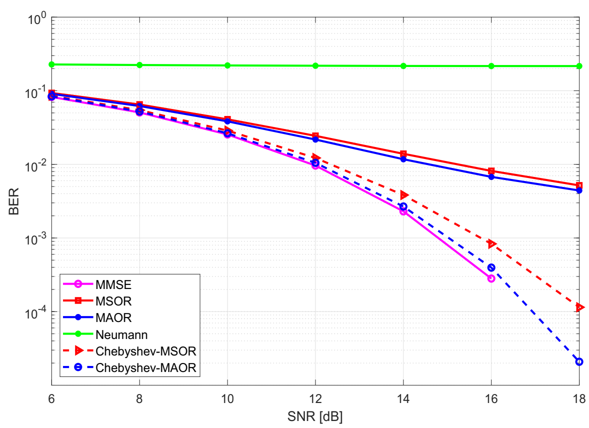
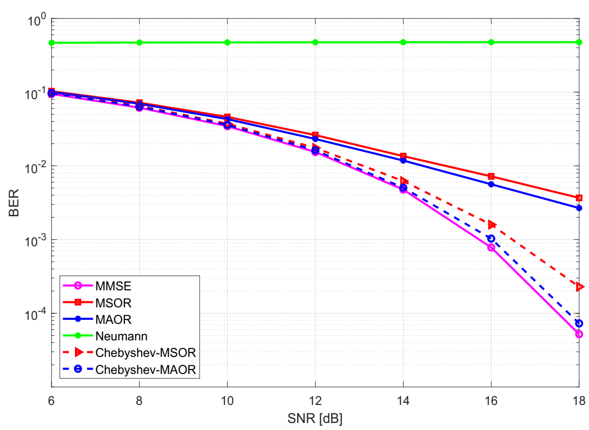


Fig. 9: BER performance for 64-QAM in a $N \times K = 256 \times 64$ scheme and i=3 iterations.



(a) $\zeta_{Rx} = 0.4$ and $i = 3$



(b) $\zeta_{Rx} = 0.6$ and $i = 5$

Fig. 10: BER performance for the BS-Correlated Channel with 64-QAM modulation and configuration antenna in a $N \times K = 128 \times 16$.

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