Frequency Domain based Method for Solving Response due to Outer Race Defect in Deep Groove Ball Bearings

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Abstract. Ball bearings play a critical role in modern industrial machines. However, localized defects, such as dents, can compromise the machine performance by increasing vibrations. In rotor dynamics, modeling and response analysis of localized defects are often carried out using nonlinear models solved with numerical integration schemes in the time domain. The time domain method can generate accurate results but the solution can become computationally expensive. This study proposes a frequency domain method for the solution of faulty ball-bearing responses. The method is known as the bearing kinematics augmented base excitation method (BKA-BEM). The study analyzes the accuracy and robustness of the proposed method in calculating responses resulting from outer race defect. The results indicate a strong level of agreement between the proposed frequency domain method and previously developed time domain methods. Furthermore, the frequency domain approach demonstrated a significant reduction in computational time, rendering it a viable option for addressing industrial optimization problems.

Keywords: Deep groove ball bearing, Fast Fourier transform, Frequency domain solution method, Localized defect

1 Introduction

Ball bearings are key components in rotating machinery. Non ideal geometry, such as localized defects in bearing outer races significantly affect the dynamic behavior of the rotating machines. Modern modeling methods are often used to predict the behavior of such systems with increasing accuracy and efficiency.

Over the past years, many researchers have worked on the modeling of localized defects in rolling bearings, almost all of which utilize time domain solution methods to solve the equation of motion. Patel et al. [1] presented an analytical model for predicting the effect of localized defects on ball-bearing vibrations. Liu et al. [2] proposed a theoretical model of a dent with a shoulder on the races of the ball bearing and analyzed the vibration behavior of the dent. Sopanen and Mikkola [3, 4] presented a 6-degrees of freedom (DOF) analytical ball bearing model that accounted for both distributed defects such as waviness of inner and outer races as well as localized defects. Atif et al. [5] developed a 6-DOF dynamic model of ball bearing to investigate the vibration characteristics due to a multi-point defect comprising a dent and bump on its raceway surface. Their proposed method is solved using 4^{th} order Runge-Kutta method. Verma and Saini [6] proposed a vibration model to analyze the defect in deep groove ball bearing. Gao et al. [7] formulated a 4-DOF dynamic model for ball bearing with multiple defects on inner and outer race and solved the equations of motion using 4^{th} order Runge-Kutta method.

While solving a nonlinear dynamic model in the time domain, a crucial limitation is computational time. Depending upon the solution of different complex parameters i.e. model complexity and time step, the computational load in the solution of equations of the system can become computationally extensive. To reduce the computation time, an alternative approach is to solve the rotor bearing model in the frequency domain. Choudhury et al. [8] developed and verified a frequency domain method for solving responses generated by a distributed defect (waviness) in a rotor-bearing system by using a 4 DOF quasi-static model to include the bearing kinematics.

In this study, the bearing kinematics and the augmented base excitation method are used to solve the response due to localized defect (dent) in the outer race of a deep groove ball-bearing. The dent is modeled using a 4-DOF bearing model to reduce the time of computation and for simplification of the system. The amplitudes and phases of the relevant spectrum components are used as a base excitation for forced displacement at the faulty bearing for the full system in frequency domain. The resulting response is compared with those from existing time domain based solution methods in terms of accuracy, robustness and computational efficiency.

The novelty of the paper is to develop methods based on frequency domain solution to model bearings with localized defect. In literature, bearing modeling are commonly solved using time domain method with numerical integration. There is limited investigation on frequency domain based solution methods for bearing models, namely [9, 8], both of which investigated bearing waviness. As per the author's knowledge, there are no studies on frequency domain based solution of bearing model with localized defect. Furthermore, since developing the bearing modeling and solution method is the focus here, fault diagnosis [10] is considered as a subsequent step and out of scope for this current study.

2 Localized Outer Race Defect Modeling

A brief description of the general approach used to solve the responses caused by localized defects i.e. dent in rolling element bearing (REB) is given below.

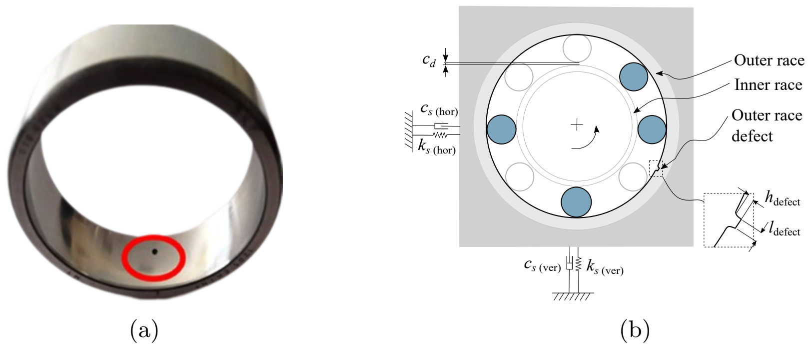


Fig. 1: (a) Outer race defect or dent in a rolling bearing [11] (b) 4-DOF bearing model where bearing kinematics from all rolling elements in contact (colored blue) is included in the response calculation

The defect in the ball bearing race affects the total deformation of the i^{th} ball which can be written as [3]

$$\delta_{\text{i}}^{\text{tot}} = 2r_{\text{b}} - d_{\text{i}} + \delta_{\text{defect}}, \tag{1}$$

where r_{b} is the radius of the ball, d_{i} is the distance between inner and outer race along the line of contact and δ_{defect} is the deformation due to the dent.

In this study, only the modeling of the outer race defect is discussed. The localized defect is assumed to be a dent that can occur due to cavitation or pitting (Fig. 1(a)). The geometry of the defect is simplified to be described by two parameters; the length (l_{defect}) and the height (h_{defect}) of the defect. A pulse is generated whenever the ball passes over the dent. A variation in the total deformation occurs when the attitude angle of ball i is equal to that of the defect on the outer race. This is expressed according to [3] as

$$\beta_{\text{i}} - \theta_{\text{in}} - \phi_{\text{defect}} = -n2\pi, n = 0, 1, ..., \tag{2}$$

where ϕ_{defect} is the position angle of the defect and β_i is the attitude agnle of ball i. In order to trigger this condition, the properties of the tangent and STEP functions can be used as in [3] to obtain deformation due to dent δ_{dent} (Eq. (1)). Using the total elastic deformation from Eq. (1), the contact force, F_i, of each ball bearing can be calculated as

$$F_i = K_{\text{c}}^{\text{tot}} (\delta_i^{\text{tot}})^{1.5}, \tag{3}$$

where $K_{\mathrm{c}}^{\mathrm{tot}}$ is the total contact stiffness of individual ball contact. The total bearing forces in lateral directions can be calculated as a sum of forces from all individual rolling elements for a bearing as [3]:

$$\mathbf{F}_b(t) = \begin{Bmatrix} F_x \\ \\ F_y \end{Bmatrix} = \begin{Bmatrix} -\sum\limits_{i=1}^{n_{re}} F_i \cos\phi_i \cos\beta_i \\ \\ -\sum\limits_{i=1}^{n_{re}} F_i \cos\phi_i \sin\beta_i \end{Bmatrix}, \tag{4}$$

where, ϕ^i is the loaded contact angle, and n_{re} is the number of balls.

3 Bearing Kinematics Augmented Base Excitation Method (BKA-BEM)

The proposed frequency domain solution method utilizes the concept of base excitation. Fig. 2 explains the steps in the method. The excitation from the localized defect in the bearing outer race is assumed to cause forced displacement between the shaft and outer race (Fig. 1 (b)). It is assumed that the outer race is rigidly connected to the bearing housing and the inner race to the rotor. The outer race defect is introduced in a 4-DOF quasi-static model to consider the bearing clearance and localized defect. The 4 DOFs include the lateral DOFs of the rotor and the support, both modeled as mass-spring-damper elements. The force from eq. 4 and gravity force vector $\mathbf{F}_g$ are included in the time domain solution of the simple 4-DOF model through numerical integration. This solution is considerably faster than that of a full rotor-bearing-support system.

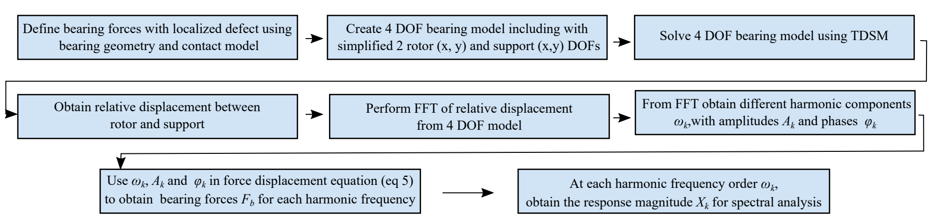


Fig. 2: Steps included in BKA-BEM modeling method

In the next step, a Fast Fourier Transform (FFT) is applied to the time domain response to identify the key frequencies and their respective amplitudes and phases. These harmonics are generated due to the dent and can be considered as base excitations or different orders acting on the rotor-bearing-support system.

The individual harmonic amplitudes and phase from the dent are considered to cause forced displacement transferred through the bearing. Therefore, the combined harmonic spring and damping forces for the k^{th} order excitation acting on the rotor are:

$$\mathbf{F}_b^k = \underbrace{(-k_b A_k \sin\phi_k - c_b A_k \omega_k \cos\phi_k)}_{\mathbf{F}_s^k} \sin\omega_k t + \underbrace{(k_b A_k \cos\phi_k - c_b A_k \omega_k \sin\phi_k)}_{\mathbf{F}_c^k} \cos\omega_k t, \tag{5}$$

where ω_k is the rotational speed times the relevant harmonics of the excitation frequency, A_k and ϕ_k are the amplitude and phase angle for k^{th} harmonic order, respectively.

At the same time, a counteracting force will act on the support with equal magnitude and in a direction opposite to the F_b. Taking into account the multiple harmonics due to dent, the equation of motion for any general rotor-bearing system can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + (\mathbf{C} + \omega\mathbf{G})\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \sum_k (\mathbf{F}_s^k \sin\omega_k t + \mathbf{F}_c^k \cos\omega_k t), \tag{6}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{K}$ are the mass, damping, gyroscopic, and stiffness matrices of the system. $\mathbf{F}_\mathrm{s}^k$ and $\mathbf{F}_\mathrm{c}^k$ are the force amplitudes due to bearing excitation. Other excitation sources such as unbalance are ignored for this study and they are not relevant for bearing fault analysis. The terms $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$, $\mathbf{x}(t)$ in Eq. (6) represent the acceleration, velocity and displacement vectors respectively. The solution of each individual harmonic excitation from Eq. (6) can be obtained using a trial solution as follows:

$$\mathbf{y}(t) = \mathbf{a}^k \sin\omega_k t + \mathbf{b}^k \cos\omega_k t, \tag{7}$$

where the coefficient vectors $\mathbf{a}^k$ and $\mathbf{b}^k$ for each k^{th} harmonic excitation are obtained as:

$$\begin{bmatrix} \mathbf{a}^k \\ \mathbf{b}^k \end{bmatrix} = \begin{bmatrix} \mathbf{K} - (\omega_k)^2\mathbf{M} & -(\omega_k)(\mathbf{C} + \omega\mathbf{G}) \\ (\omega_k)(\mathbf{C} + \omega\mathbf{G}) & \mathbf{K} - (\omega_k)^2\mathbf{M} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F}_s^k \\ \mathbf{F}_c^k \end{bmatrix}. \tag{8}$$

These individual coefficients can be used to calculate the response magnitude at frequency ω_k for the full rotor-bearing system for spectrum analysis as

$$\mathbf{X}_k = \sqrt{{a_k}^2 + {b_k}^2}. \tag{9}$$

4 Case Study

Figure 3 shows a finite element analysis (FEA) model of an electric motor from literature [4] used as a test case for the BKA-BEM modeling method. The electric motor is supported by two 6010 ball bearings. The bearing at the non-drive end

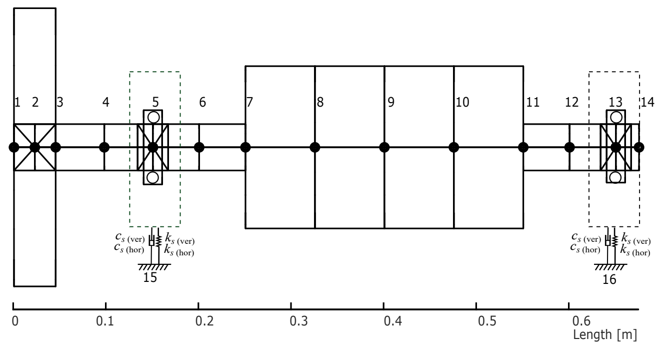


Fig. 3: FEA model of the test case electric motor with numbered elements (length is in X-direction and Y (vertical) and Z (horizontal) are the radial directions)

(NDE) is considered ideal whereas the localized dent is assumed to be in the drive end (DE) of the motor.

The simulation model utilizes 3-D beam elements based on the Timoshenko beam theory. In the FEA model, an overhung impeller is located at node 2. In the model, the bearings connect the rotor at nodes 5 and 13 to the support housing at nodes 15 and 16, respectively. The bearing stiffness and damping coefficients are obtained by linearizing the bearing forces at an equilibrium position. Table 1 includes the key properties of the bearing. A defect of 50 μm height and 2 mm length is located at a position angle of -90 deg. The support stiffness and damping are taken as 1e6 N/m and 1e4 Ns/m respectively in lateral directions. The rotor and support (bearing housing only) masses are 82 kg and 3.17 kg respectively [4].

5 Results

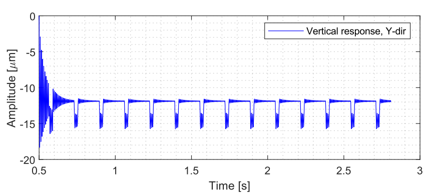
First the full rotor-bearing system is solved using time domain solution methods (TDSM) by numerical integration. The simulation uses a timestep of 5e-5 seconds for a total of 600 secs. This results in high resolution response from the model. More details about the TDSM method can be found in our previous studies [4, 8]. The FFT spectrum of the displacement response is stored as a reference for validation of the BKA-BEM method.

Next, to implement the BKA-BEM method, the quasi-static response from the bearing model is obtained. For this purpose, the 4-DOF bearing model including the dent model is simulated at 1 Hz rotational speed for one full cage rotation. The 14 balls of the bearing roll over the outer race defect causing 14 impulse peaks in the response within that time period. Figure 4 (a) and (b) show the vertical and horizontal time domain (TD) response for the faulty bearing.

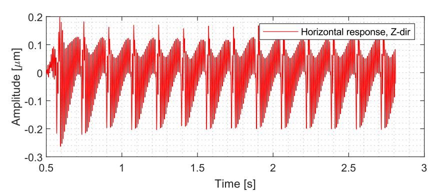
Figure 5 (a) and (b) show the vertical and horizontal amplitude and phases for the faulty bearing at the DE side. Since only the outer race defect is considered here, the outer race defect frequency f_{bpor} and its harmonics are of interest.

Table 1: Bearing parameters of the 6010 type ball bearings in the case study.

Bearing Parameters	Values
Bore diameter	50.0 mm
Outer diameter	80.0 mm
Pitch diameter	65.0 mm
Ball diameter	8.73 mm
Number of balls	14
Diametral clearance	0.059 mm
Bearing damping coefficient	0.55 Ns/mm
Inner and outer race conformity	0.52
Static load rating	16000 N
Modulus of elasticity	206000 MPa
Poisson's ratio	0.3
Pressure viscosity coefficient	0.023 mm^2/N
Absolute viscosity at zero pressure	0.04e-6 Ns/mm^2
Bearing Stiffness	
Vertical	1.18e08 N/m
Horizontal	6.90e07 N/m
Bearing Damping	
Vertical	2.95e03 Ns/m
Horizontal	1.72e03 Ns/m



(a)

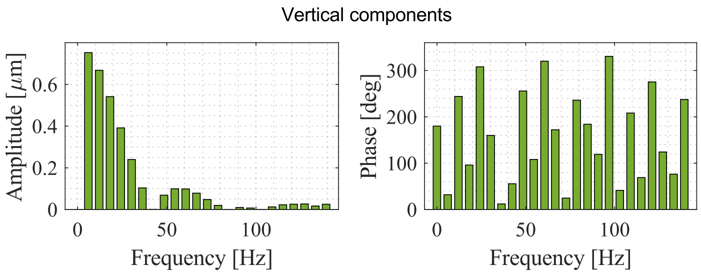


(b)

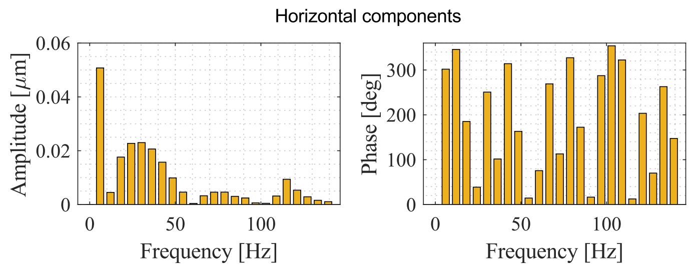
Fig. 4: Relative displacement (a) Vertical (b) Horizontal between rotor and support from a 4-DOF model with the dent in outer race

With 1 Hz rotating frequency, f_{bpor} = 6.0598 Hz. The FFT showed 683 peaks which were harmonics of f_{bpor}. However, only f_{bpor} and its 23 harmonics were considered for BKA-BEM as the rest of the peaks had very low amplitude and hence were neglected. The frequency domain solution DC amplitudes are 12.27 μm in the vertical direction and 7.6e-04 μm in the horizontal direction. The larger dc amplitude in the vertical direction is due to gravity.

Finally, Figure 6 and 7 compare the frequency spectrum of displacement obtained using TDSM and BKA-BEM in horizontal and vertical directions at DE bearing location.

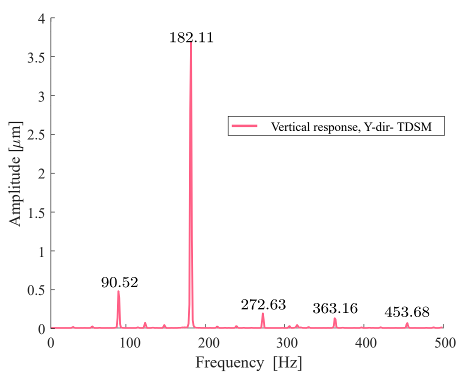


(a)

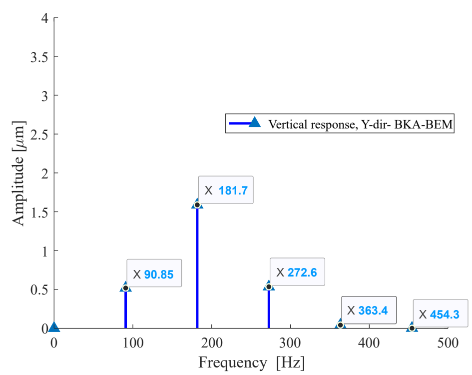


(b)

Fig. 5: Amplitude and Phases of 24 spectrum components which are harmonics of f_{bpor} (ball passing outer race frequency) in (a) vertical and (b) horizontal direction. The components with very small magnitudes are neglected.



(a)

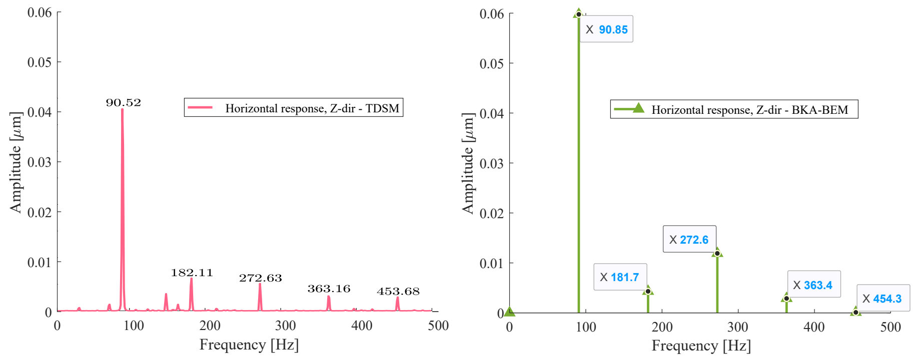


(b)

Fig. 6: Spectrum of vertical displacement response at DE bearing location (Node 5) using (a)TDSM and (b) BKA-BEM

6 Discussion

In the presented results, firstly the time domain response from the 4DOF model (Fig. 4 (a) and (b)) shows similar response for each of the 14 balls passing



(a) (b)

Fig. 7: Spectrum of Horizontal displacement response at DE bearing location (Node 5) using (a) TDSM and (b) BKA-BEM

the outer race during the single rotation. The 50 μm deep defect resulted in approximately 4 μm response in the vertical direction together with the fixed displacement due to static load and 0.5 μm response in the horizontal direction. Then in Fig. 5 (a) the overall response is broken down into their components, each with relatively smaller amplitudes compared to the time wave.

When compared with TDSM (Fig. 6 and 7), the BKA-BEM shows significantly good accuracy. The spectrum frequencies shows good match as all the peaks appear at f_{bpor} and its harmonics for both model. The amplitudes from 4-DOF model are utilized in the forced displacement equaiton in BKA-BEM for the full system. The resulting responses in vertical direction (Fig. 6) remain relatively smaller compared to the TDSM model whereas in the horizontal direction (Fig. 7), the BKA-BEM shows marginally higher response at almost all the frequencies. Using a full spectrum in the FFT could potentially address the amplitude inaccuracy since the signal resulting from the dent is not purely sinusoidal and may exhibit fluctuating amplitude across negative frequencies.

The contribution trends of the different components match remarkably well between the two models as well. In the vertical direction (Fig. 6), the 2X f_{bpor} contributes to most of the vibration response whereas in horizontal direction the f_{bpor} itself creates the most vibrations. The BKA-BEM is also able to capture the trend of relative higher vibration in vertical direction compared to horizontal. This is mainly due to the location of the dent (-90 deg from +X-axis, which means the dent is vertically located). Moreover, the BKA-BEM is especially suitable for the outer race defect simulation since there are no side-bands observed even in the TDSM response, given that the outer race remains stationary.

7 Conclusions

This study proposed a modified frequency domain method (BKA-BEM) for computationally efficient modeling and solution of localized outer race defect in ball bearing. The proposed methodology could accurately obtain the displacement response at the outer race defect frequencies and harmonics. Moreover, compared to the time domain solution method, the proposed method significantly reduced the computational time from tens of minutes to mere seconds. In future, the BKA-BEM method should be verified experimentally. The BKA-BEM can be utilized to generate discrete spectrum of a rotating machine across various speeds range under the effect of localized defect. This study has also demonstrated that the BKA-BEM method can be a rapid and effective means of generating a discrete vibration spectrum of a rotating machine across various speeds within its operational range, making it a valuable tool for quick analysis.

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