# Erratum for the Master's thesis Fractional calculus and generalised norms in condition monitoring of a load haul dumper by Juhani Nissilä

1. **p. 30, in Theorem 2.24 eq. (2.32):**

   $$\widehat{f} = \sum_{k=-\infty}^{\infty} c_k \, \delta_{2\pi k/T}. \tag{2.32}$$

   **changed into**

   $$\widehat{f} = \sum_{k=-\infty}^{\infty} c_k \, \delta_{k/T}. \tag{2.32}$$

2. **p. 40, after the proof of Theorem 3.2 added:**

   If the iterated integral ${}_{-\infty}I^m f \in L_p$ for some $m \in \mathbb{N}$, then the semigroup property with lower limit $a = -\infty$ can be extended to $\alpha + \gamma < m + 1/p$.

3. **p. 40, Theorem 3.4:**

   Let $\phi \in L_p[a, b]$, $1 \le p \le \infty$ and $\alpha, \gamma \ge 0$. Then for $f = {}_a^{RL}I^{\alpha+\gamma}\phi$

   $${}_a^{RL}D^{\alpha} \, {}_a^{RL}D^{\gamma} f = {}_a^{RL}D^{\alpha+\gamma} f. \tag{3.4}$$

   For $a = -\infty$ and $\phi \in L_p$ the Theorem holds if $\alpha + \gamma < 1/p$.

   **changed into**

   Let $\phi \in L_1[a, b]$ and $\alpha, \gamma \ge 0$. Then for $f = {}_a^{RL}I^{\alpha+\gamma}\phi$

   $${}_a^{RL}D^{\alpha} \, {}_a^{RL}D^{\gamma} f = {}_a^{RL}D^{\alpha+\gamma} f. \tag{3.4}$$

   For $a = -\infty$, $m \in \mathbb{N}$ and ${}_{-\infty}I^m \phi \in L_1$, the Theorem holds if $\alpha + \gamma < m$.

4. **p. 41, added into the end of the proof of Thm. 3.4:**

   The condition ${}_{-\infty}I^m \phi \in L_1$ for the case of $a = -\infty$ was required, because we utilised integrals ${}_a^{RL}I^{\lceil\gamma\rceil+\alpha}\phi$, ${}_aI^{\lceil\gamma\rceil}\phi$ and ${}_aI^{\lceil\alpha\rceil}\phi$ in the proof.

5. **p. 41, Theorem 3.5:**

   Let $\alpha \ge 0$. Then for all $f \in L_p[a, b]$, $1 \le p \le \infty$

   $${}_a^{RL}D^{\alpha} \, {}_a^{RL}I^{\alpha} f = f \quad \text{a.e.} \tag{3.5}$$

For $a = -\infty$ and $f \in L_p$ the Theorem holds if $\alpha < 1/p$.

**changed into**

Let $\alpha \geq 0$. Then for all $f \in L_1[a,b]$

$$ {}_a^{RL}D^\alpha \, {}_a^{RL}I^\alpha f = f \quad \text{a.e.} \tag{3.5} $$

For $a = -\infty$, $m \in \mathbb{N}$ and ${}_{-\infty}I^m f \in L_1$, the Theorem holds if $\alpha < m$.

6. **p. 42, after the proof of Theorem 3.8:**

   The assumption $\lim_{t\to\infty} {}_a^{RL}D^\alpha f(t) = 0$ is fulfilled for example if ${}_a^{RL}D^\alpha f \in L_p$, for $1 \leq p < \infty$. Actually, for fractional derivatives this assumption is not even needed, since the ordinary derivative of a constant is zero.

   **changed into**

   The assumption $\lim_{t\to\infty} {}_a^{RL}D^\alpha f(t) = 0$ is not even needed for fractional derivatives, since the ordinary derivative of a constant is zero.

7. **p. 44, after the proof of Theorem 3.10:**

   To extend the result to $\mathrm{Re}(z) \leq 1$,

   **changed into**

   To extend the result to $\mathrm{Re}(z) \geq 1$,

8. **p. 54, Theorem 4.9:**

   Let $f \in L_{1,\mathrm{loc}}$ be a $T$-periodic function and $f_T = f \cdot 1_{[0,T]}$ and $z \in \mathbb{C}$. If $\mathrm{Re}(z) < 0$ let us also assumme that $f$ has zero mean value, i.e. $c_0 = 0$. If ${}^W D^z f$ exists for all $t \in \mathbb{R}$ and ${}^F D^z f_T$ and ${}^{RL}D^z f_T$ are equal and continuous , we have

   $$ {}^F D^z f_T = {}^{RL}D^z f_T = \left({}^W D^z f\right) \cdot 1_{[0,T]}, \quad \text{for all } t \in \mathbb{R}. $$

   **changed into**

   Let $f \in L_{1,\mathrm{loc}}$ be a $T$-periodic function and $f_T = f \cdot 1_{[0,T]}$ and $z \in \mathbb{C}$. Suppose that ${}^W D^z f$ exists for all $t \in \mathbb{R}$ and ${}^F D^z f_T$ and ${}_{-\infty}^{RL}D^z f_T$ are equal and continuous. If $\mathrm{Re}(z) < 0$ let us also assume that $f$ has zero mean value, i.e. $c_0 = 0$, and $\lim_{t\to\infty} {}_{-\infty}^{RL}D^z f_T = 0$. Then

   $$ {}^F D^z f_T = {}_{-\infty}^{RL}D^z f_T = \left({}^W D^z f\right) \cdot 1_{[0,T]}, \quad \text{for all } t \in \mathbb{R}. $$

9. **p. 55, Theorem 4.10:**

Let $f \in L_1(0,T)$ and $T$-periodic and it has zero mean value, i.e. $c_k = 0$. Then

$$ {}^{W}D^{\alpha} f(t) = {}^{RL}_{-\infty}I^{\alpha} f(t), \quad 0 < \alpha < 1, $$

when ${}^{RL}_{-\infty}I^{\alpha} f$ is understood as

$$ \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{t} (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau = \frac{1}{\Gamma(\alpha)} \lim_{\substack{n\to\infty \\ n\in Z_+}} \int_{t-2n\pi}^{t} (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau $$

**changed into**

Let $f \in L_1[0,T]$ and $T$-periodic and it has zero mean value, i.e. $c_0 = 0$. Then

$$ {}^{W}D^{-\alpha} f(t) = {}^{RL}_{-\infty}I^{\alpha} f(t), \quad 0 < \alpha < 1, $$

when ${}^{RL}_{-\infty}I^{\alpha} f$ is understood as

$$ \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{t} (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau = \frac{1}{\Gamma(\alpha)} \lim_{\substack{n\to\infty \\ n\in \mathbb{N}}} \int_{t-nT}^{t} (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau $$