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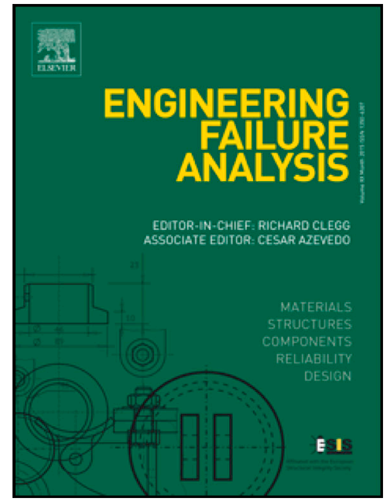

Literature review of fatigue assessment methods in residual stressed state

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Literature review of fatigue assessment methods in residual stressed state

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Abstract

Residual stresses remain in manufactured mechanical components after the forces related to the manufacturing process have been removed. Beneficial compressive residual stresses can be induced using shot peening, cold expansion of holes, and low plasticity burnishing, for example. The purpose of this review is to determine the relevant phenomena and fatigue assessment methodology of the residual stress state. It is shown that the common strategy for fatigue assessment – considering residual stresses simply as mean stresses – may lead to non-conservative predictions. Generalization of the presented methodologies is paid attention to and prospective research areas are indicated.

Keywords: Fatigue assessment, residual stress, partial crack closure

1. Introduction

Residual stresses can be found in the highly loaded notches due to service loading or thermomechanically induced plastification in hot components. Manufacturing always induces residual stresses in the material and stress relieving is an extra monetary cost. It is common knowledge that the poor fatigue performance of weldments is due to the tensile residual stresses. On the other hand, in highly stressed components additional safety or longer service life is desired. Purposely induced compressive residual stresses at the

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surface can be an option for improving the fatigue performance. The engineers assessing viability of these options and fatigue performance of residual stressed components are facing a non-trivial task.

This review aims to find out state of the art methods for assessing fatigue and crack growth in residual stress fields. These reflect both finite and infinite life design philosophies. Special attention is paid to generalization of the presented methodology to industrial and general use. The key factors and mechanisms affecting the fatigue performance, in residual stressed state, are also revisited. For the sake of simplicity, welded joints were largely excluded from the scope of this review, although most of the presented concepts and mechanisms apply for the weldments as well. For residual stress testing methods we guide the readers to the review by Withers and Bhadeshia [1]. Examples of failures related to residual stresses are given in the review by James [2]. To better help the reader follow and form an overview, discussions and summarizing is done at the end of each topic and a concluding discussion is held in the end.

2. Ways of producing residual stresses

Residual stresses originate from spatial gradient of irreversible deformation and is typically result of plastification or phase transformation. Residual stress is internally in equilibrium over the whole body. The peak magnitude of the residual stress is typically of the order of the undeformed material's yield strength [3]. In the subsection below, a few of the processes that produce residual stresses are briefly revisited.

2.1. Peening

Shot peening (SP) is the most commonly used, and most extensively studied, post-processing method to introduce compressive residual stresses and improve fatigue performance of components. In shot peening, hard spherical shots are air blasted against the surface of a component. Each impact point induces local plastic deformation on the surface. A plastically-stretched surface attempts to expand, but the adjacent elastic region restrains the expansion, creating a compressive residual stress field near the surface.

A desirable result of shot peening is the improvement in high-cycle fatigue so that crack initiation from the surface or subsurface are as probable and fatigue limit increases by approximately 10–20% compared to the base material, as shown by Torres *et al.* [4] for AISI 4340. In military combat aircraft aluminum structures, the most relevant improvement is the increase in total fatigue life, which in the best case can be several times higher in shot peened material than in an as-machined condition. Further, the fatigue limit change can be seen as secondary effect, because fatigue life is always finite in critical aircraft structures. On the other hand, Shiozowa and Lu [5] show that for a 100Cr6 bearing steel that while shot peening increased the fatigue life in the region above the surface fatigue limit of the material by changing the initiation site from surface to subsurface, the very high cycle fatigue lives initiating from subsurface were unaffected. A peculiarity for shot peened materials is that the low-cycle fatigue performance can be worse than the base material, which is attributed to the increased surface roughness, as shown by Klotz *et al.* [6] for Inconel 718. Another characteristic of shot peening is the degree of cold working. At the surface, the degree of cold working can be very high (up to 30-40%) and the gradient is very steep [7]. Consequently, the depth of the compressive residual stress layer is typically in range of 0.15–0.3 mm. The stability of the shot peening residual stresses will be discussed later. The development of numerical analysis of shot peening focuses on addressing the surface coverage which largely determines the success of the treatment [8].

2.2. Cold expansion of holes and interference fit fasteners

Cold working of holes is also a widely used post-processing method, at least in the manufacturing of aircraft structures. The basic idea of this technique is to pull a mandrel through a hole (that is larger than the hole) using a hydraulic puller. Temporary split-sleeves can be used in this process between the mandrel and the hole or permanently installed bushings [9]. In the final stage, the hole is usually reamed to ensure good surface quality. This process induces a plastic deformation that increases the hole diameter; however, the surrounding elastic region restricts this expansion, creating a compressive residual stress field around the hole. The magnitude of the peak compressive stress is roughly equal to the compressive yield stress of the material. Usually, a compressive stress region spans from one radius to one diameter from the edge of the hole [10].

Total fatigue life of the cold-worked hole is usually over three times longer than that without any post-processing [11]. This yields lighter structures when applied in design and production. The method can also be used in service life extensions or repairs in critical holes to increase fatigue life. The cold expansion process has some limitations due to high plastic deformation. The process can increase the probability of stress corrosion cracking, and if the hole ligament is too short, cold expansion can also fracture the parts during the process. Further, operational underloads can cause limitations to fatigue life improvement due to relaxation of compressive stresses [12], which is discussed later.

Another way of producing compressive stress around a hole is to use interference-fit fasteners or bushings [9]. By installing a fastener slightly larger than the hole, it is possible to create a compressive stress state around the hole. The drawback of this method is slightly more difficult installation and removal of fasteners. In spite of this, it is still widely used in aircraft structures.

2.3. Laser shock peening

Laser shock peening (LSP) utilizes a pulsed laser to generate a rapidly expanding plasma burst on the part surface. An increase in pressure generates a powerful compressive shockwave that propagates through the material, creating compressive residual stress. The plasma burst is generated from opaque overlay (tape) and transparent overlay (water), which are applied on top of the component for the process. LSP equipment is currently expensive; therefore, this method is not widely used. Compared to SP, this method produces higher compressive residual stresses and greater surface quality. One of the drawbacks of SP is the increased surface roughness. LSP, on the other hand does not face this problem. The degree of cold working is typically lower than in shot peening (about 9%). [7]

In a more recent study [13] the surface layer's microstructure was studied thoroughly. High dislocation density, dislocation entanglements, slip lines and very fine sub-grains were observed near the surface. Simulations of laser shock peening and the formed residual stresses were utilized in [14].

2.4. Low-plasticity burnishing

Low-plasticity burnishing (LBP) is a relatively new post-processing method. In this process, a ball or a wheel is hydraulically pressed against the treated surface to induce plastic deformation on the surface. The LPB process uses a fluid between the burnishing tool and the surface to avoid wearing out the tool and damaging the surface. Many series of overlapping passes are made until sufficient coverage is achieved. LPB can induce higher compressive residual stresses compared to shot peening. The process helps achieve minimized plasticity (no shearing due to slipping), which means that less cold work (about 4%) is generated at the surface. [7]

The benefits of both LPB and LSP compared to traditional shot peening are deeper compressive residual stress region with less cold working, which is illustrated in Figure 1. For high temperature components, it is important that surfaces treated by LPB or LSP have higher thermal relaxation resistance for lower cold work and lower risk in annealing or recrystallization [7]. Additionally, better improvements in fatigue performance can be achieved with these treatment methods compared to shot peening [15, 16].

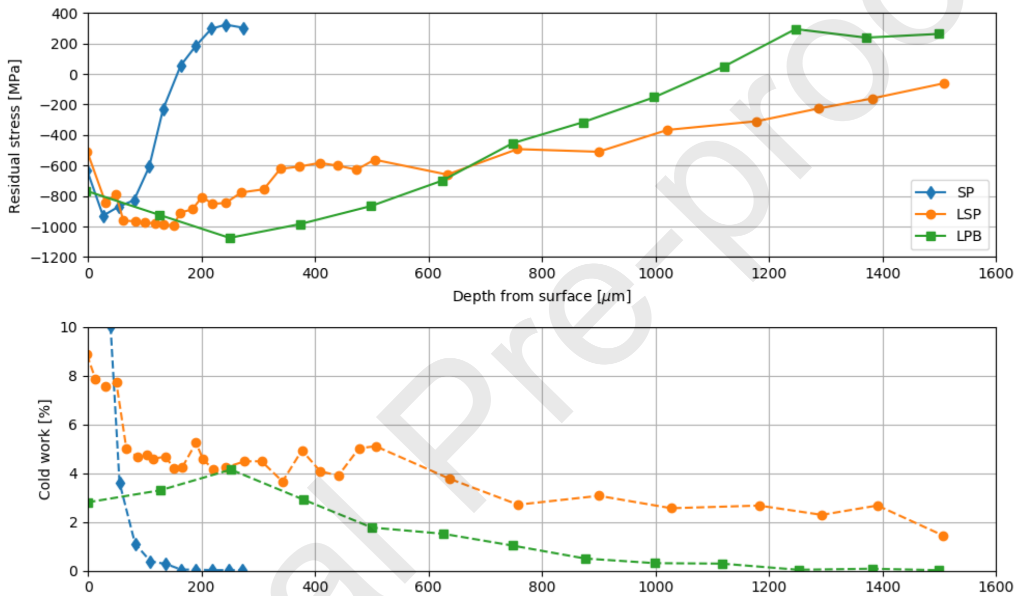

Figure 1: Residual stresses and cold work distribution in IN 718 after Shot Peening (SP), Laser Shock Peening (LSP) and Low Plasticity Burnishing (LPB). Reproduced from [7].

2.5. Manufacturing processes

There are many conventional component manufacturing processes that create residual stresses, even where that is not the primary purpose. For example, machining always produces some amount of residual stress that may be undesirable. The surface of a plain fatigue test specimen is typically electrochemically polished to eliminate unintentional effects of machining-induced residual stresses on the test results. Residual stresses can create undesired curvature in parts, which are generally relieved using heat treatment. The machining induced residual stresses and simulation prediction methods are reviewed extensively in [17] and complemented by [18, 19]. Grinding related residual stresses are reviewed in [20]. Simulation of heat treatment induced residual stresses are reviewed in [21] and welding in e.g. [22, 23]. For general overview of the role of residual stresses in fatigue of weldments we guide the readers to [24]. Numerical simulation and fatigue prediction of butt welding was performed in [25]. The residual stresses of dissimilar steel joints were considered in [26]. For more recent fatigue assessment methods the continuum damage model was applied in [27, 28], the latter considering the effect of porosity as well as residual stresses.

There are many other manufacturing processes or surface treatments that produce residual stresses and affect the fatigue life of the component, such as forging, casting, induction- and case-hardening, and deep rolling. For the sake of simplicity, in the manuscript these processes are not discussed in depth.

3. Stability of residual stresses

In the previous section, the causes of residual stresses were explained, and understanding them is crucial for understanding the mechanisms affecting the stability of residual stresses. In principle, relaxation of residual stresses always brings the neighboring material elements stress-free configurations closer, meaning it relieves the internal imbalance. This section is largely based on the extensive and recent review by McClung on the stability of residual stresses [3]. It is evident that the stability of residual stresses is crucial for the fatigue performance of components. This is highlighted in Kim *et al.*'s studies on the relaxation of residual stresses of shot-peened medium-carbon steel under rotating bending fatigue tests [29, 30]. Kim *et al.* proposed that the fatigue crack growth in their experimental scenario starts only after the residual stresses have relaxed to below 80% of their initial value. McClung in his review stated, 'complete or nearly complete relaxation of residual stresses is rare and occurs only for severe cycling, sometimes with an additional influence from elevated temperature.' McClung categorized the types of relaxations into four categories, which are elaborated in the next subsection.

3.1. Static loading

The initial residual stress tends to relax by quite a large amount during the first load cycle in operation. The amount of relaxation depends on the magnitude and direction of loading with respect to the residual stresses. The dependence on direction of loading can be explained by Bauschinger's effect [31, 32, 7, 33], where the yield surface of a plastically-deformed material translates towards the stress state that caused the irreversible deformation. The sign of residual stress in this deformed material is typically the opposite of the loading that caused it. As the yield surface has translated, yielding can be expected earlier if the load is in the opposite direction of the irreversible deformation, and later if it is to the same direction. If loaded in an opposite direction to the irreversible deformation, and yielding occurs, the internal imbalance is alleviated and the residual stresses relaxed. In other words, compressive residual stresses tend to be more stable in tensile operating loading conditions and vice-versa [3]. Occasional overloads are also considered to belong to this category. These findings were also reported by Toribio *et al.* [34, 35] with the FEA of cold-drawn wires. Stefanescu [12] showed for cold-worked holes that underloads significantly relaxed the initial compressive residual stress field. McClung notes that modeling this should be rather easy considering the knowledge available on initial residual stress state and service loading [3].

3.2. Cyclic loading

Residual stresses tend to relax with applied mechanical loading cycles. Kodama [36] measured residual stress relaxation on the surface of shot-peened specimens and proposed a model where the cyclic relaxation of residual stresses is linear with respect to the logarithmic number of cycles. The slope term depends on the applied stress amplitude. Kim *et al.* [29] documented the relaxation of shot peening-induced residual stresses thoroughly, and the corresponding data as well as their empirical model (1) is visualized in Figure 2.

Kim *et al.* proposed an empirical model to characterize the experimental data

$$\sigma_{res}(\sigma_a, N) = (1.50\sigma_Y - 2.75\sigma_a) + (-0.75\sigma_Y + 0.91\sigma_a)\log N, \qquad R = -1, \tag{1}$$

where σ_Y is the material's yield strength, σ_a is the applied stress amplitude, and N is the number of fatigue cycles. McClung notes in his review that no general model exists for modeling the relaxation of residual stresses [3]. Klotz *et al.* [6] and Kirk [37] reported initial compressive residual stress becoming tensile with high stress fatigue cycles. Wagner and Luetjering [38] concluded

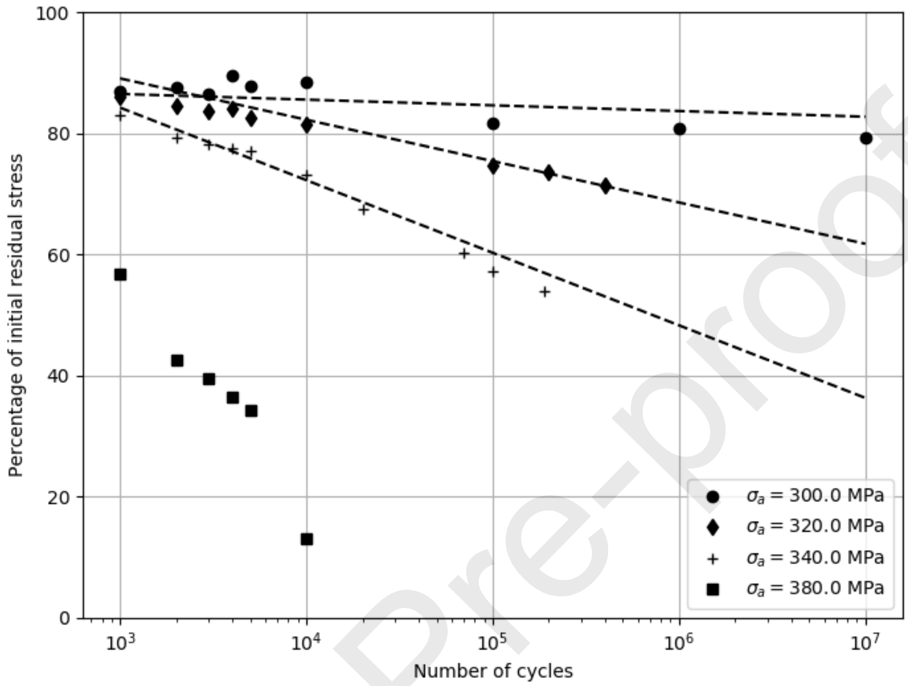

Figure 2: Percentage of initial residual stress versus number of rotating bending fatigue cycles (R=-1) in shot peened 0.45% carbon steel specimen. The dashed lines are the model (1) predictions. Reproduced from [29] using average curves.

that the cyclic stability of residual stresses depends on the material hardening or softening with the fatigue cycles.

3.3. Thermal effects

With an increase in temperature, materials tend to become softer, and the dislocation movement is easier. Creep-like mechanisms, and dislocation glide and climbing, gradually activate with temperature. Metallurgists have taken advantage of this in the process of stress relieving because it is designed to relax the induced residual stresses from manufacturing. Vöhringer *et al.* studied the relaxation of shot peening residual stresses at different annealing temperatures between 20°C and 600°C as a function of time up to 60,000 minutes for the titanium alloy Ti-6Al-4V. They found that a significant relaxation occurred at temperatures above 300°C [39]. They proposed an Avrami-type equation to describe the observed phenomena

$$\frac{\sigma_{res}(T)}{\sigma_{res}(243K)} = \exp\left[-(At)^m\right], \tag{2}$$

where the value of m is determined by the corresponding relaxation mechanism, and parameter A follows an Arrhenius-type equation

$$A = B \exp\left[\frac{-Q}{kT}\right], \tag{3}$$

where B is a constant, Q is the activation energy, k is Boltzmann's constant, and T is the absolute temperature. They fit the model to the experiments and found the effective activation energy to be $Q = 2.78$eV. They also found this value to correspond to the activation energy of α-titanium's high temperature creep and self-diffusion with a reference value of $Q = 2.51$eV, and concluded the dominating relaxation mechanism to be climbing of edge dislocations. [39]

The model (2) predictions using mean parameter values and data from [39] are shown in Figure 3.

3.4. Crack extension effects

Fukuda and Yasuyuki [40] experimentally studied the redistribution of welding residual stresses due to crack growth in JIS SS41 mild steel. They showed that the tensile residual stresses redistributed remarkably with fatigue crack propagation. They also concluded that it was indeed the crack extension, and not the cyclic loading, that was responsible for the redistribution. Lee *et al.* [41] also studied the phenomenon with mild steel SS330 and welding tensile residual stresses. They also observed the substantial redistribution of residual stresses with fatigue crack propagation. Lam and Lian [42] studied the effect of residual stress redistribution with crack extension of 2024-T3 aluminum specimen. The specimen had compressive residual stresses and no significant effect of residual stress redistribution was observed. As an explanation, they suggested that their experimental setup had low residual stresses compared to the applied external loads and that with higher magnitude of residual stresses, the effect could be more significant. Pavier *et al.* [43] studied the role of residual stresses in cold-worked holes. Even though they performed sophisticated finite element analyses that could take into account the redistribution of residual stresses, they did not draw any conclusions on

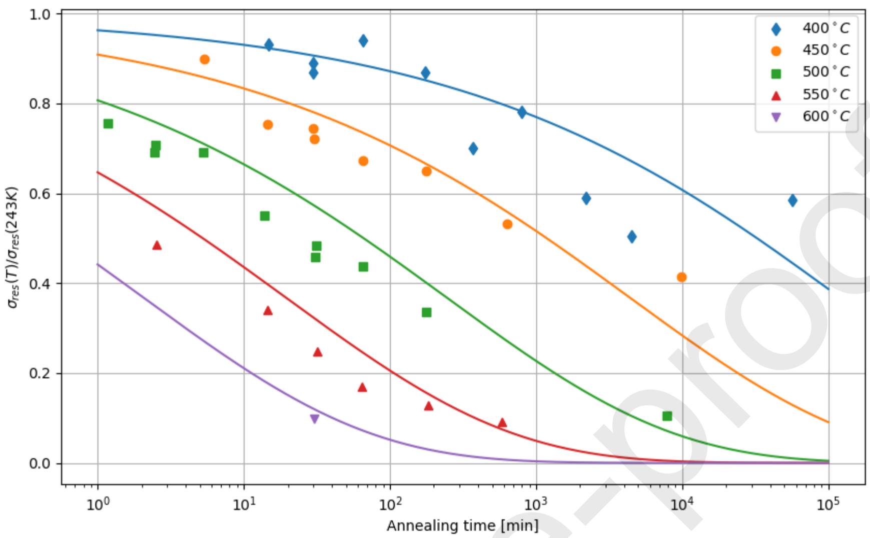

Figure 3: Percentage of initial residual stress versus annealing time at different temperatures for shot peened Ti-6Al-4V. Reproduced from [39]. Model predictions using mean parameter values.

the redistribution. However, they did demonstrate static stress distributions at different applied loading levels and introduced crack. We would like to note that a cyclic analysis, and perhaps even a crack growth scheme, should be applied to get simulation estimates. On top of this, the unloaded state of stresses should be visualized to show the effect of crack growth to the compressive residual stress redistribution.

Nelson [44] commented on the measurements by various authors in which growing the crack through an initial compressive residual stress field resulted in an increase in crack growth rate (compared to the residual stress-free measurements), but only after the crack had grown through the compressive residual stress field and most of the original tensile residual stress field. They suggested a possible explanation to this being the redistribution of the residual stresses with crack growth, which occurred when the crack extended to the tensile residual stress region. This would then effectively drop the crack opening stress levels. Nelson could have been right to predict what other researchers later suggested to be partial crack opening [31, 45] that resulted in the net increase of effective stress intensity range in situations where the crack grows in residual stress gradient.

Özdemir and Edwards [10] studied the relaxation of residual stresses of cold-worked hole on 7050-T76 aluminum alloy. At fatigue limit, they observed almost full relaxation of the surface residual stresses. Below the surface, the maximum compressive residual stress reduced from -500 MPa to -400 MPa. The residual stress profile eventually stabilized at the fatigue limit. They reported inlet side cracks arrested to approximately 1 mm in length and suggested that the relaxation was primarily due to crack growth.

Amjad *et al.* [46] recently studied experimentally the residual stress relaxation of cold-worked holes in the presence of a fatigue crack. They used thermoelastic stress analysis as well as synchrotron X-ray diffraction and concluded that the fatigue crack did not significantly relax or redistribute the compressive residual stress field by cold working.

We could not find clear measurements or simulations of significant compressive residual stress redistribution with the fatigue crack extension, given near-threshold or small-scale yielding loading levels. For tensile residual stresses in welding, these effects are more pronounced. A simple explanation would be that the introduction of a crack prevents stresses from passing through the interface in tension but does not in compression.

4. Cold-working effects on fatigue strength

Most of the methods producing residual stresses also, as a byproduct, produce cold work in the material. By cold working we mean microstructural changes due to plastic deformation or increase in dislocation density, typically indicated as either hardening or softening. Kliman *et al.* [47] collected the work of other researchers who performed fatigue tests with prestrain, i.e., a specimen without residual stresses but with microstructural changes due to cold working. The collection is shown in Figure 4, and it emphasizes the potential importance of this phenomenon. While several materials show a positive slope, there are exceptions where the fatigue limit is drastically reduced even with a small amount of prestrain. Wagner and Luetjering [38] performed interesting studies on Ti-6Al-4V, such as rotating bending fatigue tests for five different conditions: electropolished, shot-peened, shot-peened and annealed (nearly complete residual stress relief), shot-peened and electropolished (20 μm), and, finally, shot-peened, annealed, and electropolished (20 μm) specimen. The results are compiled in Table 1. The 10^7 fatigue strength for different configurations were 700, 720, 370, 845, and 800 MPa,

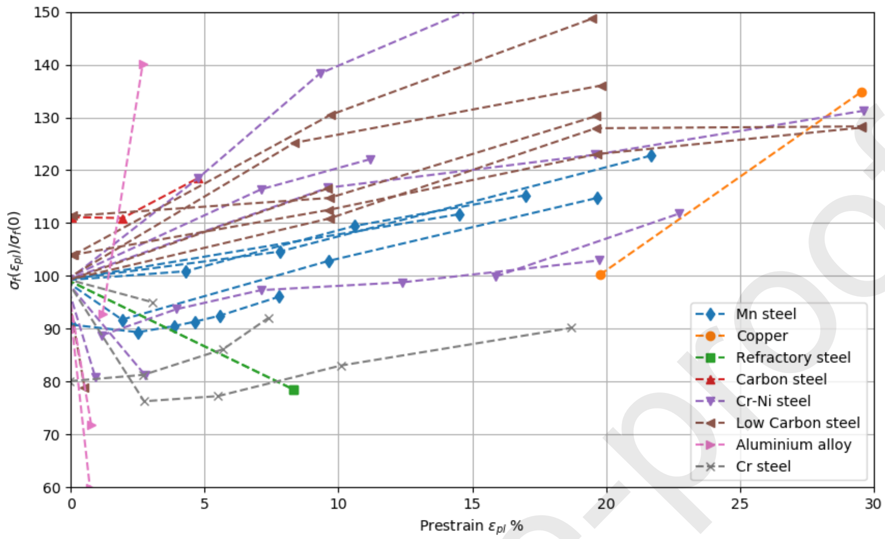

Figure 4: Relative change in fatigue limit due to cold working for various materials. Reproduced from [47] (original data from various sources).

respectively. The annealing reduced the 10^7 fatigue strength from 720 to 370 MPa, even though all of the fatigue cracks in their studies initiated from the surface. The annealed and electropolished condition regained 430 MPa of the fatigue strength to a value of 800 MPa, which is higher than for the shot-peened condition. This find was attributed to the easy initiation of cracks from the rough surface produced by shot peening. The shot-peened and electropolished surface had the highest fatigue strength. The fatigue strength of shot-peened, annealed and electropolished surface was approximately 100 MPa higher than that of only electropolished, which was deduced as being because of cold working.

The phenomena presented until now have mainly concerned the fatigue crack initiation. The following two chapters deal with phenomena concerning the fatigue crack growth emphasized in residual stressed state: crack closure and non-elliptical shaped cracks.

Table 1: The fatigue strength of Ti-6Al-4V in various surface conditions [38].

Condition	Fatigue strength [MPa]
Electropolished	700
Shot-peened	720
Shot-peened and annealed	370
Shot-peened and electropolished	845
Shot-peened, annealed and electropolished	800

5. Crack closure

The crack closure is currently seen as the dominant explanatory factor for the stress ratio dependencies in crack growth rates and threshold values. As the residual stresses can be seen to modify the effective stress ratio, a short review of the phenomenon is seen necessary here. Elber [48] noticed the compliance curve in a cracked body being nonlinear with changes in loading in elastic region – indicating varying load carrying geometry. In his Ph.D thesis, he proposed that a plastic wake is left behind a propagating fatigue crack tip that should result in a crack closure during unloading the specimen under macroscopic tensile loading. Measurements were set up to prove their hypothesis and it was concluded that, indeed, the crack in the fatigue specimen was fully open only for a part of the tensile load cycle. Previously, it was thought that closure could only occur under macroscopic compressive loading. Since then, crack closure has been under active investigation by several authors. This finding gave rise to the need for defining the opening stress of the crack to be able to outline the effective stress intensity range precisely. We shall introduce the key concepts without being too critical of the relative contributions. The readers are guided towards the recent review of related phenomenon by Pippan and Hohenwarter [49], and for near-threshold behavior the review by Suresh and Ritchie [50]. A couple of highlights are:

- Five distinct sources of crack closure were identified: plasticity, oxide, roughness, viscous fluid, and phase transformation [50]
- Crack closure is more present in plane stress conditions than in plane strain conditions [51, 52, 53]
- Physically-short crack growing from a notch does not initially exhibit crack closure, resulting in higher crack growth rates [49]

- Overload crack growth delayed retardation can be understood with temporary removal of crack closure due to crack tip blunting [49, 54]
- Finite element analyses suggest that different standard crack growth test specimen exhibit different degrees of crack closure [55]

The magnitude of crack closure can be experimentally determined using compliance techniques, crack propagation techniques, and non-mechanical contact measurements [49]. Another way of determining the amount of crack closure is through Finite Element Analysis (FEA). McClung and Sehitoglu [53, 56] wrote an early review of the modeling. Pommier and Bompard [57] studied the effect of Bauschinger effect on the plasticity-induced crack closure and concluded that there was a strong interaction between the material's cyclic plastic behavior and the observed stress ratio, overload and underload effects. A more recent work was conducted by Camas *et al.* [58]. The essential learnings of the work were on the choice of elasto-plastic constitutive model, crack advancing scheme, and mesh refinement compared to crack growth rate and size of plastic zone.

Pommier *et al.* [59] reported sensitivity of the crack opening load to the minimum load at the negative stress ratios for N18 superalloy, and the detrimental effect of high compression on the fatigue crack growth rates. The results are shown in Figure 5. It can be seen that the crack opening load was found to be negative for high applied compression. When the magnitude of the compression reduced at $R = -1$, the crack opening load became similar to that measured at $R \approx 0$. They reached a good agreement with the measured opening levels with FEA. Silva [60] reported the crack closure concept as being inadequate in explaining the observed crack growth rates at $R = -1$. They performed fatigue crack growth tests for various materials and found that as the maximum applied load increased, the opening load decreased; however, for some materials the crack growth rate did not change accordingly. For cyclically hardening materials, the crack growth rate increased with the increase of maximum load (which was the case for Pommier), whereas for cyclically softening materials, the reverse occurred. For materials that exhibit neither cyclic hardening nor softening, the crack growth rates were insensitive to the load amplitude. Using the sizes of monotonic and cyclic plastic zones of Rice [61], they rationalized that for negative load ratios the role of cyclic plastic material behavior was emphasized. They then suggested that the materials' varying degrees of Bauschinger effect could

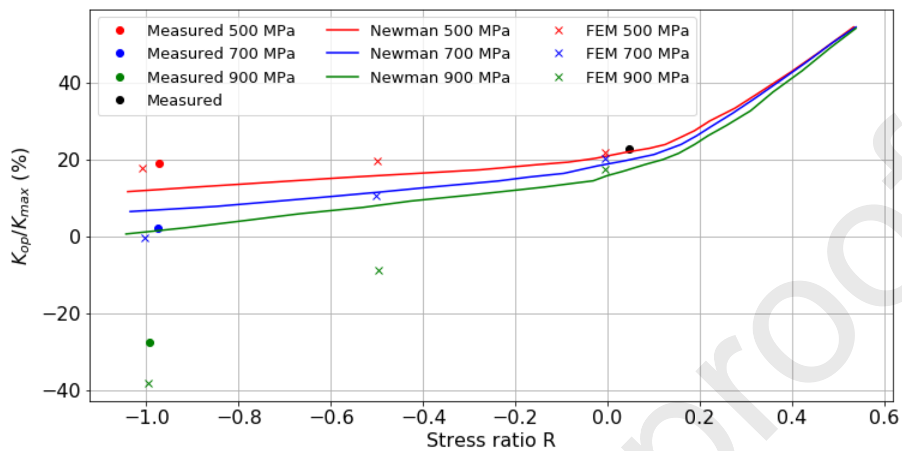

Figure 5: Evolution of the estimated crack opening stress intensity factor as function of stress ratio and maximum stress. Reproduced from [59].

explain the phenomenon; high Bauschinger effect would cause cyclic plastification ahead of the crack tip during compression and relax the effective compressive residual stresses. Thus, the compression at $R = -1$ would be detrimental to the materials exhibiting high Bauschinger effect. We would like to note that based on the measurements of Pommier, and the explanation offered, it may be safe to assume that the importance of this effect is diminished at load levels near crack growth threshold. So, an analogy to the lack of cyclic residual stress relaxation at fatigue limit can be inferred here.

The studies performed on fatigue crack growth retardation due to overloads are interesting in light of residual stresses. The role of plasticity-induced crack closure and residual stresses in overload related crack growth retardation is still under active discussion [54, 62]. Jones [63] studied the strain-hardening effect on un-notched fatigue crack growth of Ti-6Al-4V. Understanding that annealed material hardens with plastic deformation, his original hypothesis was that the overload retardation effect could be partly explained by strain hardening. The strain hardening, however, increased the crack growth rate and could not thus explain the overload retardation. Robin *et al.* [64] performed experimental studies on single-overload fatigue crack retardation and concluded that the effective stress intensity factor due to crack closure, measured by compliance method, could not produce the observed transient crack growth behavior, when the crack growth rate was recovering from the overload retardation. They also found that a better agreement with the experimental data could be reached by calculating residual stresses due to the overload. Shercliff and Fleck [65] reached similar conclusions regarding the assessment based on crack opening stress after overload being non-conservative. Based on FEA, they suggested the reason for the discrepancy was the partial closure of the crack. Paris *et al.* [66] proposed a simple model for partial crack closure modifications to the effective stress intensity range that was later modified by Borrego *et al.* [67] to provide a smoother transition from the overload as a function of crack length.

Salvati *et al.* [54] attempted to separate the effects of crack closure and residual stresses after an overload by testing first at high load ratio (0.7) to induce crack growth without crack closure, which was verified by in-situ Digital Image Correlation (DIC). Following the overload the strain distribution was quantified with synchrotron to calibrate an elastoplastic FE-model. Tests with lower load ratio (0.1) were also performed and crack closure was observed before the overload and to a greater extent after the overload. The crack growth rate of the closure-free test (R=0.7) quickly returned (after half of the overload plastic zone size) to the pre-overload steady-state values. With the lower load ratio test (R=0.1), where closure was present, the crack growth retardation effect after the overload lasted longer. It was then inferred from the experimental crack growth rates, calibrated FE-model and in-situ DIC measurements, that the two effects were similar in magnitude but the closure contributed over a longer distance after the overload. Thielen *et al.* [62] studied the near crack tip stress fields of overloaded and baseline crack growth specimens with in-situ synchrotron and concluded that directly after the overload the residual stresses have dominant role in explaining the crack growth behavior.

For negative applied stress ratios, Halliday *et al.* [68], Makabe *et al.* [69] and Silva [70] reported an overall acceleration of crack growth after overload, as opposed to the expectation of retardation, for certain materials, both in plane stress and plane strain. Halliday considered short cracks whereas Makabe and Silva considered long cracks. FEA performed by Halliday could predict the changes in residual stresses ahead of the crack tip, which was in agreement with the observed crack growth behavior. Silva found for certain materials almost no effect of overloads or underloads at negative base load ratios, and also concluded that the materials' cyclic plastic properties—Bauschinger effect in particular—seemed to control the crack growth behavior at negative stress ratios. Silva emphasized that the crack closure, or any other proposed mechanism, could not explain the observed behavior, and suggested that, although crack closure could explain most features of the fatigue crack growth, it should be considered more as a consequence rather than a cause. It should be noted that as Silva and Pommier previously reported the sensitivity of the crack opening loads to the maximum compression (or load amplitude) at negative stress ratios, no consideration was made for the effect of base load amplitude on the phenomenon here. Given the explanation of cyclic plasticity, the crack growth rate should naturally, as in the case of non-overload negative stress ratio findings, approach the findings of $R = 0$ with decreasing load amplitudes.

Suresh and Ritchie [50] argued that the intrinsic crack length dependence due to crack closure breaks the similitude concept of fracture mechanics. They discouraged the interpretation of fracture mechanical data using nominal ΔK-based concepts due to loss of uniqueness, not representing the true crack driving force. Their proposed solution was to develop analyses capable of capturing the mechanics of fatigue crack: cyclic plasticity, non-stationary crack tip fields, and crack closure. *"Until such analyses are available, the use of ΔK_{eff}, representing closure-adjusted ΔK values, provides probably the most fundamental approach, at least for academic assessment of fatigue behavior"*, they concluded. Vasudevan and Sadananda have also raised concern on focusing what is happening behind the crack tip instead of the crack tip internal stresses (see e.g. [71]). Vasudevan *et al.* [72] and Suresh [50] have criticized the difficulties of determining unique crack closure levels.

6. Non-elliptical shaped cracks

Pell *et al.* [73] studied crack growth rate from a cold-worked hole made in aluminum alloy. They noticed that without cold working, the cracks were elliptical. They also noticed a difference in the mandrel entry and exit face crack lengths. The crack had grown into a shape what they called "bulbous nose". The crack depth on entry and exit side of cold-expanded (C-E) hole as a number of flights, corresponding loading spectrum repetitions, are shown in Figure 6. The control group had no cold-expansion treatment and the crack growth can be found to be several times faster. Kokaly *et al.* [74] made similar observations for cold-worked holes. They noticed that with increased thickness of the plate, the relative difference of crack growth rates between the entry and exit faces increased. They performed FEA to analyze

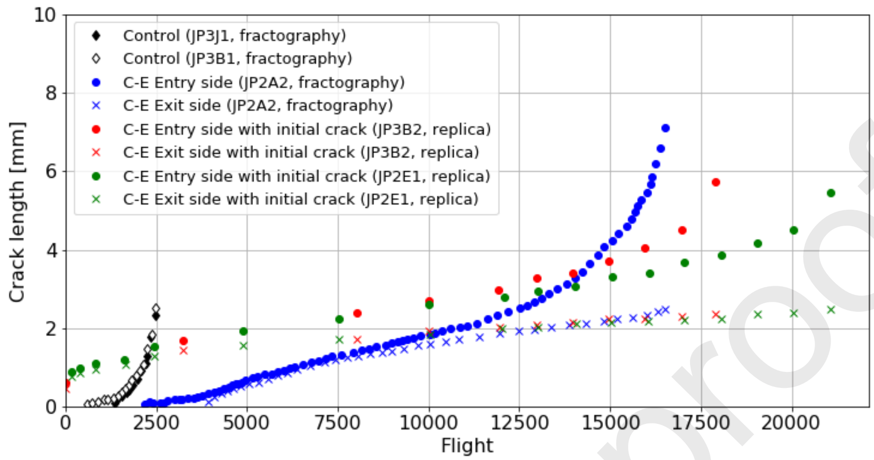

Figure 6: Crack growth on entry and exit sides of a hole subjected to aircraft loading spectrum repetitions. Reproduced from [73].

the residual stress state and an analysis on crack growth, and found that the non-symmetric distribution of the residual stresses in the radial direction could explain the phenomenon. McClung [3] commented on the finding by Prevéy *et al.* [15] in which they made an artificial semi-circular crack using electrical discharge machining (EDM) for IN718 alloy and grew the crack in fatigue up to certain length, after which they applied low plasticity burnishing on the surface. The crack that grew after the treatment was found to be of a peculiar shape. Liu *et al.* [75] used a cohesive zone model-based approach to study in 3D the crack propagation in a shot-peened specimen. They also could observe non-elliptical cracks in their simulation with the presence of the residual stresses. They, however, could not produce experimental verification for their model. Many authors above argued for the need for improved analysis capabilities beyond just assuming the elliptical crack shapes in these situations.

7. Fatigue assessment

Thus far, we have discussed ways of producing residual stresses, the stability of residual stresses, and effects of cold working for the fatigue strength. Browsing through literature, many of these phenomena were studied separately. The majority of the studies are experimental in nature, and little effort went into the predictions that would assist the practical engineering work. Depending on the end destination for the component with residual stress treatment, either stress life (SN-curve) or fatigue crack growth rate are typically measured. These two schools are also present in making the fatigue assessments; crack initiation and fracture mechanical crack growth approaches. The former typically aims to study changes in the fatigue limit (infinite life design philosophy) whereas the latter aims to primarily study changes in the fatigue crack growth rate (finite life design philosophy).

7.1. Fatigue crack initiation

Most of the approaches aimed for assessing fatigue crack initiation treat the residual stresses as mean stresses. Much like in fatigue, no general workflow seems to overshadow the others, but instead, every researcher uses their favored fatigue criteria. Let us next describe some of the fatigue assessment methods proposed for residual stressed states. We shall only consider publications where the fatigue assessment was performed with a goal of predicting or reflecting to the observed fatigue behavior. The prediction error used here is the relative difference of the predicted and measured values. The models are not explicitly written to avoid the vast amount of parameters to be introduced. Instead, the workflows are described and the reader is given the references to follow for further reading.

Leitner *et al.* [76] recently performed induction hardening simulations for 50CrMo4 to simulate the induced residual stress field. The fatigue assessment was performed on basis of linear-elastic stress analysis combined with the Ramberg–Osgood relationship to correct the strain amplitude from the linear-elastic stress amplitude. After this, they combined the Smith, Watson and Topper (SWT) damage parameter [77] with the total strain life formula by Manson, Coffin and Basquin. The damage parameter considers maximum principal stress and principal strain range on the maximum principal strain range plane. The parameters were derived from the local hardness measurements and uniform material law described in [78]. They performed fatigue tests using notched specimen and got a decent match with the fatigue life prediction. However, the model could not capture the measured fatigue limit. No consideration was made for the redistribution of the residual stresses.

Kliman *et al.* [47, 79] described a workflow for finding the optimal parameters for the cold-worked hole. Their model is based on Haigh diagram and identifying changes that can be applied to it. They considered microstructural changes due to cold working as shown in Figure 4, changes in surface roughness and surface properties, residual stresses acting as mean stress, material's sensitivity to mean stress (i.e. slope of the Haigh diagram), cyclic material properties, and secondary factors such as strain ageing. They did point out the uncertainty with the assumption of the cold-working effects measured for uniaxially stressed specimen, because most of the real surface residual stress methods yield biaxial stretch (and thus biaxial residual stresses). They verified the proposed model with aluminum alloy Al-Cu-Mg, for which the cold working had a negative effect on fatigue limit. They were able to estimate optimal residual strain in good agreement with the measured values.

Fathallah *et al.* [80] studied the high-cycle fatigue prediction using Crossland and Dang Van criteria [81] for shot-peened SAE 3415 notched flat three-point bending specimen. The stress ratio used in the tests was $R = 0.1$. They modeled the shot peening defects (called superficial damage in the paper) using the principles of continuum damage mechanics by Lemaitre [82] to lower the fatigue limit at the surface. A modification to the fatigue limit was applied to account for cold working as a power-law function of the ratio of the width of the diffraction peak at half the maximum value of diffraction (FWHM), which is commonly considered to be a measure of the amount of cold work. The work-hardening coefficient was chosen as the exponent of the power-law measured from the tensile test mechanical response. They noted that the cold working correction factor was very close to unity for this material due to the work-hardening coefficient. The stress concentration due to surface imperfections of the successive shot peening indentations was simulated using FEA based on the shot radius and exposure time. The fatigue limits were calculated using the as-machined fatigue tests and the fatigue limit at stress ratio $R = -1$ was estimated using the empirical relationship given by Gerber. The superficial damage was calibrated to the measured fatigue strengths under different peening conditions and fatigue criteria. We would like to comment that without the use of this superficial damage, the prediction errors increase to 6.4%. Furthermore, choosing the Goodman diagram to predict the fatigue limit at stress ratio $R = -1$, instead of Gerber, causes the prediction error to rise to 85.9%. The choice of Gerber's mean stress correction here effectively suppresses the mean stress sensitivity. The ratio of fatigue limit at stress ratio $R = -1$ and ultimate tensile strength using the Gerber diagram is 0.29, which is low for mild steel (typically values in the range of 0.4-0.5 should be expected, see e.g. [83, Chapter 6, p. 291]) (ultimate tensile strength was reported to be 510 MPa). Had the fatigue limit at stress ratio $R \leq -1$ been measured, the described assessment would have been more assuring.

Fernandez Pariente and Guagliano [84] performed rotating bending fatigue tests for 42CrMo4 steel with three different surface treatment conditions: gas nitrided, gas nitrided + shot-peened, and gas nitrided + shot-peened + partly stress-relieved. They introduced a small artificial crack in the specimen in an attempt to control the initiation site. The residual stresses were measured in each condition. They also measured the micro-hardness distributions along the depth of the material. All of these were measured both before and after the test using run-out specimen. Following Murakami [85], they applied the common fracture mechanics-based approach and found that only the gas-nitrided prediction was inside 10% error margin (predicted 13.9 vs measured 14.2 MPam$^{1/2}$). The prediction errors for the other two conditions were approximately -35%. They argued that the measured micro-hardness could not describe the cold-working effects of shot peening because no substantial difference could be observed between the nitrided and nitrided + shot-peened conditions. On the other hand, the FHWM values were different for different treatments. Inspired by the work of [80] they then extended Murakami's prediction formula to take into account the changes in the surface FWHM values. This correction provided a vastly enhanced prediction capability as the prediction errors were within 2% error margin.

Albizuri *et al.* [86] performed thorough measurements for 34CrNiMo6-QT steel in machined, polished, shot-peened and low plasticity burnished conditions. The fatigue performance was measured with $R = -1$ rotating bending tests where relaxation of the residual stresses and surface finishing quality were measured. The fatigue limit improved by 39% from the machined for the shot-peened specimen and 52% for the LPB'd specimen, whereas the polished specimen reached similar fatigue performance with the shot-peened specimen. They used von Mises effective stable residual stress and Dietmann's mean stress fitting criterion to reach agreement with the observed fatigue limit improvements for the LPB treatment and did not include cold working effects.

Bagherifard *et al.* [87] combined Taylor's theory of critical distances (TCD) [88] with the Sines fatigue criterion [89] to predict the fatigue limit of shot-peened notched and smooth 40NiCrMo7 specimen in rotating bending, and notched specimens in axial fatigue tests. The axial fatigue tests were performed using load ratio $R = 0.1$ and the mean stress sensitivity parameter was calibrated to these two load ratio results. They further modified the fatigue limits based on the measured surface roughness [90] and, based on the measured FHWM values, the cold working hardening correction, as described in [84]. They noticed that using the measured biaxial residual stress as the mean stress (hydrostatic stress) resulted in significantly higher prediction error (23.7% on average for rotating bending tests) compared to treating the compressive residual stresses as uniaxial in the cyclic stress amplitude direction (-5.5% on average for rotating bending tests and 3.9% for the axial tests). The biaxial residual stress with Sines criterion resulted in non-conservative predictions. The reason for this was not elaborated. In the same paper, they used fracture mechanics-based Atzori approach, described in [91]. For this approach, they approximated the fatigue limit mean-stress sensitivity using Morrow approximation. The prediction errors were on an average -16.8% for the axial tests, overestimating the differences between the different peening conditions and resulting in overly conservative predictions. The reason for this was thought to be the relaxation of residual stresses. We would like to comment that the measured differences in the FWHM values, and thus the cold working effect, between the peened and un-peened conditions were practically nonexistent, unlike for [84], especially considering the unreported measurement uncertainty.

In the second part of the study by Bagherifard *et al.* [92], a local fatigue limit concept by Eichlseder [93] considering the stress gradient was extended with surface-roughness correction and cold-working effect. The prediction errors with this procedure were on an average -16% for the rotating bending tests and 9% for the axial tests. They also performed calculations according to the FKM guidelines described in [94] using both the nominal and local stress approaches. The nominal stress approach had an average prediction error of -4.4% for the rotating bending tests and -4.5% for the axial tests. The local stress approach in turn had average prediction errors of -5.6% and -10.2%, respectively.

Gerin *et al.* [95, 96] studied fatigue behavior of forged surface with various surface conditions. The residual stresses due to shot blasting and shot peening were measured. The surface profiles were scanned, fatigue loading modeled using elastic FEM and the fatigue performance analyzed using Dang Van fatigue criterion combined with TCD. The Dang Van parameters were fit to give the best overall agreement with the experimental results. The prediction results were roughly contained within ±15% error margins.

The approaches are compiled in Table 2. Some researchers used the very surface stresses of shot peening indentations analyzed by FEA, whereas oth-

Authors	Specimen	Material	Fatigue criterion	Conditions	Relaxation	Cold-working	Surface roughness	Mean stress	Notes
[76]	Notched Four-point bending	50CrMo4	SWT	Induction hardened	No	Yes	No	Uniaxial $\sigma_{\max} = \sigma_a + \sigma_{\text{res}}$	Crack initiation [illegible]neath the hardened [illegible] for high-cycle fatigu[illegible]
[47, 79]	Cold hole expansion	Al-Cu-Mg	Haigh diagram	Cold hole expansion	No, discussed	Literature	No	Uniaxial Haigh diagram	Workflow to opti[illegible] process parameters [illegible]ented on idea level
[80]	Notched Three-point bending R=0.1	SAE 3415	Crossland Dang Van	Machined Shot-peened	Neglected	FWHM	SEM idealized FEM	Hydrostatic Gerber	Great extrapol[illegible] in mean stress a[illegible] witnessed
[84]	Sandglass Rotating bending Artificial micro-hole	42CrMo4	Murakami	Nitrided Nitrided+Shot-peened Nitrided+Shot-peened +Partly stress relieved	Measured	FWHM	Artificial micro-hole	Murakami Effective R	Measured HV c[illegible] not explain the obser[illegible] cold-working effects.
[86]	Smooth Rotating bending	34CrNiMo6	Basquin	Machined Polished shot-peened low-plasticity burnished	Measured	No	Measured	Von Mises Dietmann	Model for residu[illegible]stress relaxation
[87, 92]	Notched and smooth Rotating bending Axial R=0.1	40NiCrMo7	Sines+TCD Atzori Eichlseder FKM	Machined Shot-peened	Neglected	FWHM	Measured Literature	Hydrostatic Uniaxial Measured Morrow	Hydrostatic [illegible]mean stress resulted [illegible] non-conservative esti[illegible]ons
[95, 96]	Smooth flat Plane bending Axial R=-1	C70	Dang Van+TCD	Polished stress-relieved as forged forged + shot-peened forged + shot-blasted	Neglected	Neglected	Scanned FEM/Peterson	Hydrostatic	Fatigue criterion [illegible]ameters best fit to all [illegible]surements

Table 2: Compilation of fatigue assessment approaches in the literature.

ers were driven towards less local approaches, such as TCD or approaches that include a stress gradient correction factor. Some of these criteria use hydrostatic stress as a measure of mean stress with varying success, whereas other criteria only considered the residual stress in the direction of the uniaxial loading. The mean stress sensitivity of the material's fatigue strength is naturally pronounced, and it would be preferable to measure the fatigue strength at compressive mean stresses instead of extrapolating with one of the classic mean stress models. Majority of the studies neglected relaxation or redistribution of the residual stresses, especially when predicting fatigue limit was the goal. Cold working effects were generally accounted for; however, only one of the studies mentioned above used measured data for the effects on fatigue limit. The other approaches to take into account the cold working were largely phenomenological and perhaps specific to the material. The Vickers hardness in Murakami's formula could not capture the cold working effects but the FWHM could. Cold working generally increases the surface roughness, and models capturing these effects were widely used. We would like to note that this emphasizes the complexity of the phenomenon, and not a single study was found where all of the ingredients were systematically measured and then combined to make a fatigue prediction model. Until then, uncertainty will always be present, and no generalization of the approach can be made to account for different materials and differences between the test specimen and the real components.

7.2. Fatigue crack growth rate

The traditional engineering fatigue analysis procedure has two main phases: 1) crack initiation analysis using stress- or strain-life-based methods and 2) crack growth analysis using fracture mechanics-based methods. After the initiation of the fatigue crack, the interest naturally shifts to the questions – how critical is the crack, how fast does the crack grow, and will it stop? In the presence of residual stresses, some interesting phenomena in crack growth, such as emphasized crack closure, partial crack opening, and non-elliptical-shaped cracks, were reported.

In the following subsection, we discuss some of the most common strategies for fracture mechanics-based crack-growth prediction methods.

7.2.1. Superposition principle

The superposition principle of linear elastic fracture mechanics (LEFM) has been utilized for its simplicity. In superposition principle, complex loading is divided into simpler loadings that have known stress intensity factor solutions, and the contribution of each load ingredient to the stress intensity factor, analyzed separately, is summed up. Another key concept is Bueckner's weight functions [97], which allows analyzing any kind of loading once the weight functions for the geometry are known. The analysis in residual stressed state typically uses weight functions to integrate over the crack flank for the effect of residual stresses on the stress intensity factor K_{I}^{R} at the crack tip, as shown in (4). We follow Parker [98] in the description of the analysis process

$$K_{\mathrm{I}}^{R} = \int_{a} p(x)m(x,a)dx, \tag{4}$$

where $p(x)$ is the residual stress acting on the crack line of an un-cracked body and $m(x,a)$ is the weight function. Then, the effective stress intensity range ΔK and load ratio R are determined. For the case where the minimum stress intensity factor due to the external load $K_{\mathrm{I_{min}}}^{L}$ added to the residual stress intensity factor is positive ($K_{\mathrm{I_{min}}}^{L} + K_{\mathrm{I}}^{R} > 0$), we get:

$$\Delta K = K_{\mathrm{I_{max}}}^{L} - K_{\mathrm{I_{min}}}^{L} \tag{5}$$

$$R = \frac{K_{\mathrm{I_{min}}}^{L} + K_{\mathrm{I}}^{R}}{K_{\mathrm{I_{max}}}^{L} + K_{\mathrm{I}}^{R}} \tag{6}$$

And for the case where the minimum stress intensity factor is negative ($K_{\mathrm{I_{min}}}^{L} + K_{\mathrm{I}}^{R} \leq 0$)

$$\Delta K = K_{\mathrm{I_{max}}}^{L} + K_{\mathrm{I}}^{R} \tag{7}$$

$$R = 0 \tag{8}$$

Effective R-method involves measuring the crack growth curves with different R-ratios, and either having the parameters interpolated or using some model taking into account the changes in R-ratio. Parker [98] found that, in the case of partially closed crack, a check was to ensure that the crack flank displacement field was non-overlapping

$$v(x,a) = \frac{2}{H} \int K_{\mathrm{I}}(a)m(x,a)da, \tag{9}$$

where H is an elastic constant depending on whether the study is in plane-strain or plane-stress conditions. If nonphysical overlapping is found, then a nonlinear contact pressure acting on the crack flanks should be iterated until the crack flanks do not overlap anymore. Although the contact pressure is dependent on the displacement of the crack flanks, it does not violate the principles of superposition. Todoroki and Kobayashi [99] showed this methodology in action and found that its predictions were in good agreement with the FEA model as well as the measurements for S35C steel. Beghini and Bertini [100] came to similar conclusions with a C-Mn steel. In more recent analyses, FEM has been utilized to account for the contact of the crack flanks as well as possible residual stress redistribution with the crack growth [101]. Good agreement with measurements was reached for LSP-treated AA2024-T3 CT-specimen's crack growth rates in [14] where the crack growth rates were fit to the unpeened crack growth rate. Pavan *et al.* [101] performed crack growth tests for AA2524-T351 aluminium alloy middle-crack tension specimen with and without LSP-treatment. Similar to Keller *et al.* [14], the prediction based on LEFM, superposition principle and rigid contact of the crack faces in FE-analysis yielded the best agreement with the measurements. All above mentioned successful use of superposition principle include through-plate crack configuration.

Concerns have been raised over the validity of the superposition principles not accounting for residual stress redistribution as the crack propagates through the residual stress field, which may result in non-conservative predictions [41, 42, 43]. The answer to this has been the use of FEM to naturally include possible redistribution of the residual stresses [101]. Some authors have suggested that superposition could not take into account partial closure of the crack [102], which, as pointed out by Parker [98], is not true. The reader is suggested to read the lengthy discussion between Nelson and Parker [103] on arguments presented in Nelson's article [44]. The proposed limitation of $R \geq 0$ set by Parker clearly do not take into account the more recent findings on negative applied stress ratios [59, 57, 60, 70].

To take into account the overload-related crack retardation effects without macroscopic residual stresses, empirical models by Wheeler [104] or Willenborg [105] are commonly used. According to these models, retardation occurs while the crack tip plastic zone is within the overload plastic zone. The Willenborg model predicts a residual stress intensity factor generated by the overload, and utilizes it in the superposition principle to define an effective stress ratio. Although we could not find use of these models in macroscopic residual stress fields, in principle, if the overload modifications to the original residual stress field can be approximated, then the workflow described above should yield results that are in agreement with the principles of Willenborg model.

Based on the findings, we note that the superposition principle combined with the weight functions is an effective tool for analyzing the stress intensity factors of residual stresses. More recent analyses utilize FEM for accounting for the contact of crack flanks and possible redistribution of the residual stresses. Typically, in practical engineering scenarios, only mode I loading with one-dimensional residual stress field is used. However, the weight functions can be used for modes II and III as well. Weight functions generally do exist for 3D, but are restricted to elliptical-shaped cracks.

7.2.2. Models for crack closure and findings in residual stress fields

The crack closure approach attempts to capture the crack opening/closing stresses in order to get a physically-accurate effective stress intensity factor range that drives crack growth. Crack opening stress is a function of the effective stress ratio R and can be defined experimentally or numerically, as discussed earlier. The effective stress intensity range then becomes:

$$\Delta K_{\mathrm{I_{eff}}} = K_{\mathrm{I_{max}}} - K_{\mathrm{I_{op}}}, \tag{10}$$

where $K_{\mathrm{I_{op}}}$ is the crack opening stress intensity factor. The main idea is that there exist intrinsic parameters that form a master curve for the crack growth, and the conditions at the crack tip vary due to crack closure. Characterizing the crack closure and using $\Delta K_{\mathrm{I_{eff}}}$ should then fall to the master curve under all conditions. The crack closure approach has been considered superior for crack growth analysis in weldments [3]. It can also be used more flexibly to analyze effects of load history [44]. Based on his measurements on 2024-T3 aluminum alloy, Elber [106] proposed an empirical relationship between the ratio of the closure intensity factor and maximum intensity factor as a function of load ratio. Modifications to this relationship have been proposed by various other authors. Newman [52] developed an analytical Dugdale-type model to calculate the crack opening stresses, and found that the predictions of crack growth rates were in good agreement with the experimental data. Later, on the basis of FE analyses, Newman provided an empirical model to predict the crack opening stress [107], which was popularized in NASGRO model by Forman and Mettu [108]. Pommier *et al.* [59] proposed modification to the Newman's model to cover the load level dependence at negative stress ratios. The difference in crack opening predictions was attributed to the differences in the constitutive modeling. Newman used a perfectly plastic material model in his analyses, which was unable to capture the Bauschinger effect.

In macroscopic residual stress fields, Mukai *et al.* [109] reported that a crack growing through a compressive residual stress field to the tensile residual stress field cannot be explained with the conventional crack opening load. A compliance curve with an unusual shape, where the unloading and loading paths were different, was observed by several authors [45, 31], and they suggested partial crack closing behavior as the cause of this observation. A workflow was proposed to define the effective stress intensity range based on the partial crack closure stress intensity $K_{\mathrm{I}_{\mathrm{part,op}}}$, which is defined from the measured compliance curve [109]. The measurement data from Kang *et al.* [45] is visualized in Figure 7. Kang *et al.* [45] found a good agreement between their measurements and this method. Choi and Song [31] performed FEA to simulate the effect. They replicated the asymmetric loading and unloading behavior of the crack tip opening and closing, but found that the simulated crack mouth closing value was in good agreement with the measured partial opening value.

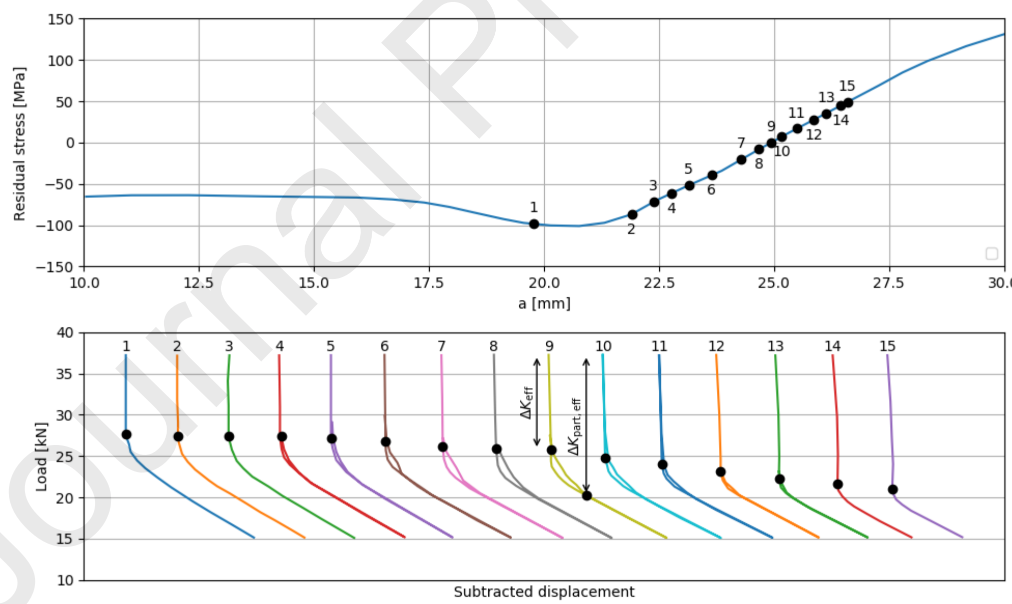

Figure 7: Load versus subtracted displacement curves in the residual stress transition zone. Conventional crack opening load is highlighted as along with the proposed partial opening load for curve 9. Reproduced from [45].

To summarize, crack closure aims to describe the physical reason for the crack growth stress ratio effects. Residual stresses naturally affect the crack opening stresses, and complex behavior of partial crack opening is reasoned to occur when the crack propagates from a compressive residual stress field to a tensile residual stress field. Similar phenomena were observed after tensile overload without macroscopic residual stresses. Generalization of the crack growth assessment to different geometries and in 3D appears to be possible only via numerical analyses (FEM). No studies were found where the crack closure were modeled to take into account the crack tip cyclic conditions and combined with strain energy release rate -based criterion (from FEA) to correlate the measurements. The crack growth threshold values in residual stress fields have been paid little attention. Interestingly, the researchers studying crack growth in residual stressed fields have not cited the research conducted on overloads, and they have seemingly independently reached similar conclusions regarding partial crack closure. The proposed analysis methodology is different for these two groups of researchers. The cyclic plastic behavior of the material ahead of crack tip is more pronounced at negative stress ratios in the form of compressive residual stress relaxation. In these situations, the combined effect of residual stresses and crack closure seems to be rather complex, and no simple generalization can be made. Under macroscopic compressive residual stresses, superposition effectively produces negative stress ratios even with positive applied stress ratios. It is not clear, however, whether the effects observed for the negative applied stress ratios apply in the case where applied stress ratio is positive but due to compressive residual stresses the effective stress ratio is negative. Because of the lack of experimental evidence for significant compressive residual stress relaxation due to crack extension, with positive applied load ratio, it would be reasonable to assume that they do not apply.

8. Discussion

A lot of the practical engineering works with residual stresses utilizes empirically-measured values for fatigue strength improvement or fatigue crack growth rate reduction, more or less directly from the test specimen to the component assessment/validation. Reading the literature quickly reveals that fatigue assessment in the residual stressed state can be complicated. This knowledge is crucial when deciding, for example, whether to try to enhance the component's fatigue properties by taking advantage of the residual stresses. General guidelines can be given for eliminating the need for more accurate consideration of the residual stresses - like when the service load amplitude clearly causes cyclic plasticity or in a case where the component is exposed to significantly elevated temperatures. In these situations, the role of residual stresses is diminished, and other means for improving the fatigue performance should be considered. Other than those conditions, it is easy to realize that the problem consists of many parts that must be either measured or assumed.

The relaxation of the residual stresses could certainly be captured with state-of-the-art constitutive material models. These models have successfully been used to simulate the static relaxation and effects of crack extension on residual stresses. The calculations for cyclic relaxation can get computationally expensive, and thus, the role of empirical models in capturing the effects holds. There are, however, a few solver techniques for accelerating the cyclic development in FEA that could be utilized for this purpose. For infinite life approximations, bearing in mind the very high cycle fatigue, the relaxation typically stabilizes, and modeling the static relaxation will suffice.

When only finite life is of interest, the problem can be simplified by only considering the macroscopic crack growth phase. Research groups working on fracture mechanics have focused their studies largely on the effects of residual stresses on the fatigue crack growth rate instead of threshold values. The principle of superposition and the weight functions can be used to get rapid estimates. Due to the nature of crack closure, confidence on calculations increases in situations where the measurement scenario is closer to that of the scenario of the real component. The generalization of the methodology always involves measuring the relaxation of residual stresses as well as the crack opening behavior when performing crack growth tests. Furthermore, to get general estimates of the crack closure, sophisticated FEA is required. In these analyses, the cyclic plastic material behavior should be captured with the crack growth scheme and refinement of the mesh. For predictions, crack closure is an essential parameter yielding the physical reason for R-ratio dependencies, which are pronounced in the presence of residual stresses and notches. The prediction is based on finding the crack opening stress/load to define the minimum load, and then calculating the effective stress intensity factor range based on that load. In the FEA of partial crack opening, which is suggested to occur after tensile overload and in residual stress gradients, the mouth closing levels were found to better correlate with the measured crack opening levels. It is not clear, however, whether in a simulation capable of capturing the crack closure, it would be more straight-forward to use a strain energy release rate-based criterion. FE analyses seem to answer the need for analyses that can take into account the physical phenomena and derive values closer to the real crack tip driving force, as described by Suresh and Ritchie. Presently, FEA is used only to correlate the measured crack opening stresses, and its full potential in analyzing the crack tip stress history has not been realized yet. The findings at negative stress ratios also emphasize the complex relationship of crack closure, residual stresses, and material's cyclic plastic behavior, which were successfully captured with FEM. Multiaxiality of the residual stresses is typically ignored, and an analysis is performed in the first-mode direction. The role of cold working effects in macroscopic residual stress fields have not been widely considered in crack growth approaches, even though the effects on crack initiation and crack growth rate have been successfully shown with measurements. It would be natural to presume that cold working would alter crack growth curves from non-cold worked, as shown by Jones. The observed non-elliptical cracks in the presence of residual stresses are a clear scenario where the weight function methodology, restricted largely to elliptical cracks, cannot predict accurately.

Some fracture mechanics researchers tend to emphasize the importance of keeping the analysis simple. It seems rather difficult to find simple general tools that could serve engineers and researchers alike in fatigue assessment in residual stressed states. Numerical methods (FEM) have been successfully used to analyze these complex phenomena for over 40 years. Increasing the detail in these analyses tend to yield better predictions. The fundamental property that needs to be measured is the material's cyclic plastic properties, which are distinct from those found in the crack growth tests (i.e. can be measured separately and is repeatable). With this, FEM is then capable of delivering estimates for both the residual stresses and plasticity-induced crack closure for cracks of any shape and loading scenario. The pursuit to arrive at one parameter/mechanism (as in crack closure) to explain all the associated phenomena has not been beneficial to the development of analysis tools, especially given the difficulties faced in uniquely determining crack closure values and corner cases where compliance based crack opening load does not yield an explanation for the observed phenomena. A typical counter-argument is that *good enough* estimates can be achieved with these simple tools, which is certainly true, especially for engineering works combined with corresponding safety factors. However, for scientific progress in this field and for the advancement of analysis methodology it will be more productive to focus on developing numerical models capable of capturing the distinct physical phenomena observed in these situations. As a more concrete example, the oxide-induced crack closure could be modeled as a time- and loading-dependent evolution equation-based mechanical description of the contact interface. Development of this kind of model then naturally gives rise to questions regarding model parameters and ways to test them. It is our view that the less we have to infer from the experimental crack growth curves, the better. Ideally, the model parameters could be tested separately, like in the case of cyclic plasticity.

If a more precise total fatigue life or infinite life prediction is needed, then the crack initiation phase should be analyzed. Unfortunately, we found no studies that base their prediction on systematically measured aspects of the phenomenon: relaxation of residual stresses, changes in surface roughness, effects of cold working, base material fatigue properties, and mean stress sensitivity. Presently, there is no generally accepted fatigue assessment criterion. Here, the role of multiaxiality of the residual stresses again seems to be unclear; typically, similar to crack growth, only the residual stresses acting in the direction of the loading were considered, compromising the validity of multiaxial fatigue criteria using hydrostatic stress as a measure for compressive mean stress. The hydrostatic mean stress could result in the overestimation of the positive effect of compressive residual stresses, paving the way for non-conservative estimates. Hardness measured locally for the cold-worked layers did not prove to be explanatory for the changes in fatigue strength due to cold-working. Extending the methodology to predicting arbitrary components and states again requires the characterization of the material's cyclic plastic behavior and initialization/simulation of the material state due to the residual stress-generating process to capture the Bauschinger's effect properly. Treating residual stresses simply as mean stresses provides a crude estimate for practical engineering purposes. Neglecting the relaxation of residual stresses, effects of cold work, and modifications to surface roughness can lead to non-conservative estimates and as a consequence to an increase in required safety factors in the absence of the quantification of these effects.

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- Hydrostatic compressive residual stress might result in non-conservative estimates
- Cold-working effects on fatigue strength are typically neglected
- Numerical methods have improved estimates of crack closure and residual stresses