Rate Maximization for Partially Connected Hybrid Beamforming in Single-User MIMO Systems

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***Abstract*—Partially connected hybrid beamforming (HBF) is a promising approach to alleviate the implementation of large scale millimeter-wave multiple-input multiple-output (MIMO) systems. In this paper, we develop rate maximizing algorithms for the full array- and subarray-based processing strategies of partially connected HBF. We formulate the rate maximization problem as a weighted mean square error minimization problem and use alternating optimization to tackle it. Numerical results show that partially connected HBF provides a good balance between hardware complexity and performance in comparison to optimal fully digital and analog beamforming. Moreover, the simpler subarray-based HBF algorithm achieves comparable performance to that of the full array-based approach in medium and high SNRs. The rate maximizing results serve as upper bounds for lower complexity heuristic methods.**

***Index Terms*—hybrid precoding, massive MIMO, mm-wave communications, rate maximization, weighted minimum mean square error.**

I. Introduction

Properly designed millimeter wave (mm-wave) massive multiple-input multiple-output (MIMO) techniques can significantly increase capacity of the next generation mobile networks [1], [2]. Implementing a fully digital mm-wave massive MIMO beamforming system with one radio frequency (RF) chain per antenna is impractical due to the high number of required power hungry RF components [2]. Hybrid digital-analog beamforming (HBF) is a promising solution to reduce the hardware complexity of large-scale MIMO systems [2], [3]. HBF can be categorized into fully and partially connected RF architectures. In a fully connected architecture, each RF chain is connected to all antenna elements while in a partially connected design, each RF chain is connected to a subarray of antennas. Partially connected design is more practical due to its lower hardware complexity [2], [4]. The HBF process of partially connected architecture can be divided into full array- and subarray-based processing strategies. In a full array-based processing design, all data streams are communicated to all subarrays while in a subarray-based processing strategy, each data stream is communicated to only one subarray [5]. The former approach can exploit all antennas for beamforming of each stream. However, partial connectivity sets restrictions on beam directions. In the latter design, each subarray is dedicated to transmit only one stream. Thus, beam directions are flexible but array gain is limited by the number of antennas at each subarray.

HBF has gained a lot of research interest in recent years [3]–[12]. Various heuristic and optimization-based approaches have been considered for partially connected HBF [4]–[9]. Authors in [7] proposed an iterative alternating optimization-based algorithm where the optimal digital precoder is approximated by a feasible hybrid precoder based on the minimum mean square error (MMSE) principle. In [8], authors developed a HBF solution for the considered rate maximization problem by approximating it as a single-stream beamforming problem with per-antenna power constraints. A novel HBF design with phase shifter selection was studied for large antenna arrays in [9]. Most of the HBF works in the literature consider only phase shifters in the analog part of the beamforming process. Employing also analog amplitude control improves the performance of HBF although with an increased implementation cost [2]. Only few algorithms have been proposed for partially connected HBF employing both analog amplitude and phase control [4]–[6]. Authors in [4] proposed an algorithm that minimizes the mean square error (MSE) between the optimal precoder and hybrid beamforming matrix. In [5], several heuristic algorithms were developed including singular value decomposition matching, iterative orthogonalization, and transmit-receive zero-forcing. Different optimization-based hybrid methods were devised for frequency selective MIMO systems in [6]. In the literature, there is still a need for research on optimization-based rate maximizing algorithms that consider both full array- and subarray-based HBF designs in order to study achievable rate upper bounds and compare the performance of the two approaches.

In this paper, we propose two rate maximizing HBF algorithms for single-user (SU)-MIMO systems with partially connected RF architecture. The algorithms are designed for full array- and subarray-based HBF processing strategies. In addition to phase shifting, amplitude control is also employed in the analog part of the HBF process. The considered rate maximization problem is equivalently reformulated as a weighted MSE minimization problem. This problem is solved by alternately optimizing between the receive combiner, the digital precoder and the analog beamformer until the objective value converges. Since the original problem is non-convex, the optimality of the solution can not be guaranteed. Numerical

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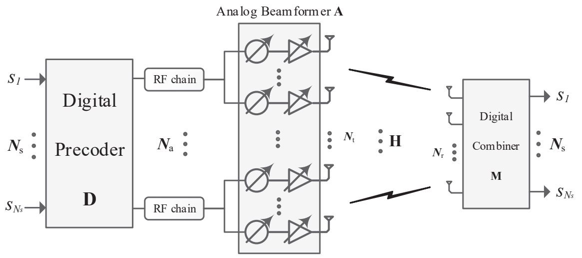


Fig. 1: MIMO system with transmitter side HBF

simulations are conducted to evaluate the performance of the proposed HBF algorithms against optimal fully digital and analog beamforming solutions.

II. System Model

Consider the downlink of a SU-MIMO system in which the base station (BS) is equipped with N_t transmit antennas and the user has N_r receive antennas. The number of receive antennas is assumed to be considerably smaller than the number of transmit antennas. Thus, HBF is considered only in the BS and the user is assumed to employ digital beamforming. Moreover, partially connected architecture is used in which each RF chain is connected to only one subarray of the antennas. The BS has N_a RF chains and the transmit antenna array is partitioned into N_a subarrays each with $n = N_t/N_a$ antennas. The number of data streams is N_s which we assume to be equal to the number of RF chains and less than or equal to the number of receive antennas, i.e., $N_s = N_a \leq N_r$. Fig. 1 depicts the overall system model with partially connected hybrid architecture at the BS in which amplitude control is employed in addition to phase shifting in the analog domain. Figs. 2a and 2b illustrate the two different processing strategies that have been considered in partially connected HBF. The received signal vector at the user is given by

$$\mathbf{y} = \mathbf{HADs} + \mathbf{z} = \sum_{j=1}^{N_a} \mathbf{H}_j \mathbf{a}_j \mathbf{d}_j \mathbf{s} + \mathbf{z} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$, $\mathbf{A} \in \mathbb{C}^{N_t \times N_a}$, and $\mathbf{D} \in \mathbb{C}^{N_a \times N_s}$ denote the channel matrix, the analog RF beamformer, and the digital beamforming matrix, respectively. Moreover, $\mathbf{s} = (s_1, s_2, \ldots, s_{N_s})^T \in \mathbb{C}^{N_s \times 1}$ is the vector of data streams with $\mathbb{E}[\mathbf{ss}^H] = \mathbf{I}_{N_s}$, and $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_{N_r})$ stands for additive white Gaussian noise. The frequency flat channel matrix can be written as $\mathbf{H} = (\mathbf{H}_1\ \mathbf{H}_2\ \ldots\ \mathbf{H}_{N_a})$ where sub-channel $\mathbf{H}_j \in \mathbb{C}^{N_r \times n}$ is the matrix of complex channel gains between transmit antennas of the jth subarray and N_r receive antennas. In the case of full array-based processing, where all data streams are connected to all RF chains, the digital precoder is given by

$$\mathbf{D} = \begin{pmatrix} d_{11} & d_{12} & \ldots & d_{1N_s} \\ d_{21} & d_{22} & \ldots & d_{2N_s} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N_a 1} & d_{N_a 2} & \ldots & d_{N_a N_s} \end{pmatrix} = \begin{pmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{N_a} \end{pmatrix} \tag{2}$$

where $\mathbf{d}_j = (\, d_{j1}\, d_{j2} \ldots d_{jN_s} \,)$ is the jth row vector of the digital precoder. In the case of subarray-based processing, where each data stream is connected to only one RF chain, the digital precoder becomes a diagonal matrix $\mathbf{D} = \mathrm{diag}\,(\, d_{11}\, d_{22}\, \ldots\, d_{N_a N_s} \,)$. The digital weights can be directly incorporated into the analog beamformer amplitudes and phases of the corresponding subarray. Thus, the digital precoder can be normalized to be identity matrix which only routes data streams to the RF chains. The analog beamformer can be expressed as

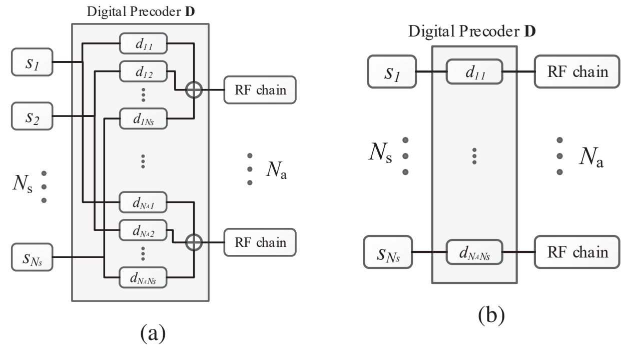


Fig. 2: (a) Full array- and (b) subarray-based processing strategies for partially connected HBF.

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{a}_{N_a} \end{pmatrix} \tag{3}$$

where $\mathbf{a}_j \in \mathbb{C}^{n \times 1}$ is the analog RF beamformer of the jth subarray, and $\mathbf{0} \in \mathbb{C}^{n \times 1}$ is a zero vector.

The achievable rate of the corresponding system is given by

$$R = \log_2 |\mathbf{I} + \frac{1}{N_0} \mathbf{D}^H \mathbf{A}^H \mathbf{H}^H \mathbf{M} \mathbf{M}^H \mathbf{HAD}| \tag{4}$$

where $\mathbf{M} = (\, \mathbf{m}_1\ \mathbf{m}_2\ \ldots\ \mathbf{m}_{N_s} \,) \in \mathbb{C}^{N_r \times N_s}$ denotes the digital combiner. The vector $\mathbf{m}_i \in \mathbb{C}^{N_r \times 1}$ is the ith combiner of the corresponding spatial data stream. The rate of stream i can be written as

$$R_i = \log_2 |1 + \mathbf{v}_i^H \mathbf{H}^H \mathbf{m}_i q_i^{-1} \mathbf{m}_i^H \mathbf{H} \mathbf{v}_i| \tag{5}$$

in which

$$q_i = \mathbf{m}_i^H \mathbf{H} \sum_{\substack{l=1 \\ l \neq i}}^{N_s} \mathbf{v}_l \mathbf{v}_l^H \mathbf{H}^H \mathbf{m}_i + N_0 \mathbf{m}_i^H \mathbf{m}_i \tag{6}$$

and $\mathbf{v}_i = (\mathbf{a}_1 d_{1i},\ \mathbf{a}_2 d_{2i},\ \ldots,\ \mathbf{a}_{N_a} d_{N_a i})^T \in \mathbb{C}^{N_t \times 1}$ is the overall hybrid precoder of stream i.

III. Problem Formulation

The optimization objective in the considered SU-MIMO system is to maximize the rate of the user while satisfying the maximum transmission power constraint. This rate maximization problem is written as

$$\begin{aligned} &\underset{\mathbf{A},\mathbf{D},\mathbf{M}}{\text{maximize}} \sum_{i=1}^{N_s} R_i \\ &\text{s.t.} \quad \mathrm{tr}\,(\mathbf{ADD}^H \mathbf{A}^H) \leq P. \end{aligned} \tag{7}$$

where P is the maximum transmission power at the BS. Solving (7) requires digital precoder $\mathbf{D}$, analog beamformer $\mathbf{A}$, and receive combiner $\mathbf{M}$ to be optimized jointly. However, this joint optimization problem is non-convex and cannot be optimally solved in its current form. In the following, we reformulate (7) as a weighted MMSE (WMMSE) optimization problem. This problem is still non-convex but it can be suboptimally solved by using an iterative alternating optimization method.

The error matrix at the output of the combiner is given by

$$\begin{aligned}\mathbf{E} &= \mathbb{E}\left[\left(\mathbf{s}-\mathbf{M}^H\mathbf{y}\right)\left(\mathbf{s}-\mathbf{M}^H\mathbf{y}\right)^H\right]\\ &= \mathbf{I}-\mathbf{M}^H\mathbf{HAD}-\mathbf{D}^H\mathbf{A}^H\mathbf{H}^H\mathbf{M}+N_0\mathbf{M}^H\mathbf{M}\\ &\quad+\mathbf{M}^H\mathbf{HADD}^H\mathbf{A}^H\mathbf{H}^H\mathbf{M}.\end{aligned} \tag{8}$$

The rate optimal MMSE combiner can be derived as

$$\mathbf{M} = (\mathbf{HADD}^H\mathbf{A}^H\mathbf{H}^H + N_0\mathbf{I}_{N_r})^{-1}\mathbf{HAD}. \tag{9}$$

The error matrix after applying the combiner is given by

$$\mathbf{E} = (\mathbf{I} + \frac{1}{N_0}\mathbf{D}^H\mathbf{A}^H\mathbf{H}^H\mathbf{HAD})^{-1} \tag{10}$$

and the error term corresponding to stream i is

$$e_i = (1 + \mathbf{v}_i^H\mathbf{H}^H\mathbf{Q}_i^{-1}\mathbf{H}\mathbf{v}_i)^{-1}. \tag{11}$$

where

$$\mathbf{Q}_i = \mathbf{H}\sum_{\substack{l=1\\ l\neq i}}^{N_s}\mathbf{v}_l\mathbf{v}_l^H\mathbf{H}^H + N_0\mathbf{I}. \tag{12}$$

It can be shown that $R = \sum_{i=1}^{N_s} R_i = \log_2|\mathbf{E}^{-1}| = \sum_{i=1}^{N_s}\log_2|e_i^{-1}|$. Authors in [13] showed that by successive approximation of the objective function, the sum rate maximization problem for fully digital beamforming can be iteratively solved via WMMSE optimization. For fixed approximation coefficients (weights), our corresponding HBF optimization problem can be written as

$$\begin{aligned}&\underset{\mathbf{A},\mathbf{D},\mathbf{M}}{\text{minimize}}\ \mathrm{tr}\,(\mathbf{WE})\\ &\text{s.t.}\quad \mathrm{tr}\,(\mathbf{ADD}^H\mathbf{A}^H) \leq P\end{aligned} \tag{13}$$

or equivalently

$$\begin{aligned}&\underset{\{\mathbf{a}_i\},\{\mathbf{d}_i\},\{\mathbf{m}_i\}}{\text{minimize}}\ \sum_{i=1}^{N_s} w_i e_i\\ &\text{s.t.}\quad \sum_{j=1}^{N_a}\mathrm{tr}\,(\mathbf{a}_j\mathbf{d}_j\mathbf{d}_j^H\mathbf{a}_j^H) \leq P\end{aligned} \tag{14}$$

where $\mathbf{W} = \mathrm{diag}\,(\,w_1\,w_2\,\ldots\,w_{N_s}\,)$ is the weight matrix and

$$w_i = e_i^{-1} \tag{15}$$

is the weight of stream i in which e_i belongs to the previous iteration. In the following, we propose HBF algorithms to suboptimally solve the non-convex WMMSE problem using alternating optimization over the receive combiner, digital precoder, and analog beamformer.

Algorithm 1 Full Array-Based Hybrid WMMSE Algorithm

Set iteration number $n = 0$ and initialize $\mathbf{D}^n$ and $\mathbf{A}^n$.
repeat
  Update $n = n + 1$.
  Solve (9) for $\mathbf{M}^n$ while $\mathbf{D}^{n-1}$ and $\mathbf{A}^{n-1}$ are fixed.
  Compute $\mathbf{W}^n$ from (15), (18) given $\mathbf{D}^{n-1}$, $\mathbf{A}^{n-1}$, and $\mathbf{M}^n$.
  Solve (17) for $\mathbf{D}^n$ while $\mathbf{M}^n$ and $\mathbf{A}^{n-1}$ are fixed.
  Solve (20) for $\{\mathbf{a}_j^n\}$ while $\mathbf{M}^n$ and $\mathbf{D}^n$ are fixed.
until desired level of convergence

IV. HBF Algorithms

In this section, two rate maximization algorithms are developed for full array- and subarray-based processing strategies of partially connected HBF architecture. These algorithms employ alternating optimization to solve (13).

A. Full Array-Based Hybrid WMMSE Algorithm

In this algorithm, the aim is to solve the WMMSE problem by using an iterative alternating optimization method where the problem is solved for one variable while others are fixed at each step. This algorithm consists of three main steps. First, (13) is solved with respect to the receive combiner $\mathbf{M}$ while the digital and analog beamformers are fixed. Then, the analog beamformer $\mathbf{A}$ and the combiner $\mathbf{M}$ are kept fixed and (13) is solved for the digital precoder $\mathbf{D}$. Last step is to optimize the analog beamformer $\mathbf{A}$ while keeping the other two variables fixed. In the following, the hybrid WMMSE algorithm is described in detail.

Problem (13) is convex with respect to the receive combiner $\mathbf{M}$. The Lagrangian expression of (13) is given by

$$\mathcal{L} = \mathrm{tr}\,(\mathbf{WE}) + \alpha(\mathrm{tr}\,(\mathbf{ADD}^H\mathbf{A}^H) - P) \tag{16}$$

where α is the Lagrange multiplier. The first order optimality condition of (16) yields the MMSE combiner $\mathbf{M}$ in (9). The next step is to solve (13) for $\mathbf{D}$. The resulting expression from the first order optimality condition is

$$\mathbf{D} = (\mathbf{A}^H\mathbf{H}^H\mathbf{MWM}^H\mathbf{HA} + \alpha\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{H}^H\mathbf{MW} \tag{17}$$

where $\alpha \geq 0$ is chosen such that the transmit power constraint is satisfied. If $\alpha = 0$ satisfies the transmit power constraint, the closed form solution is ready. Otherwise $\alpha > 0$ can be found using one dimensional search techniques such as the bisection method. In the next step, we fix the digital precoder and the combiner and solve (14) with respect to the analog beamformers for different subarrays. Rewriting the error term corresponding with stream i yields

$$\begin{aligned}e_i &= 1 - \mathbf{m}_i^H\sum_{j=1}^{N_a}\mathbf{H}_j\mathbf{a}_j d_{ji} - \sum_{j=1}^{N_a} d_{ji}^*\mathbf{a}_j^H\mathbf{H}_j^H\mathbf{m}_i\\ &\quad+\mathbf{m}_i^H\sum_{j=1}^{N_a}\mathbf{H}_j\mathbf{a}_j\mathbf{d}_j\sum_{k=1}^{N_a}\mathbf{d}_k^H\mathbf{a}_k^H\mathbf{H}_k^H\mathbf{m}_i + N_0\mathbf{m}_i^H\mathbf{m}_i.\end{aligned} \tag{18}$$

Consequently, the Lagrangian expression corresponding to (14) can be expressed as

$$\mathcal{L} = \sum_{i=1}^{N_s} w_i e_i + \alpha(\sum_{j=1}^{N_a}\mathrm{tr}\,(\mathbf{a}_j\mathbf{d}_j\mathbf{d}_j^H\mathbf{a}_j^H) - P). \tag{19}$$

$$\mathbf{a}_j = (\mathbf{H}_j^H \mathbf{M}\mathbf{W}\mathbf{M}^H \mathbf{H}_j + \alpha \mathbf{I}_n)^{-1} \mathbf{H}_j^H \Bigg(\sum_{i=1}^{N_s} \mathbf{m}_i w_i d_{ji}^* - \mathbf{M}\mathbf{W}\mathbf{M}^H \sum_{\substack{k=1 \\ k\neq j}}^{N_a} \mathbf{H}_k \mathbf{a}_k \mathbf{d}_k \mathbf{d}_j^H \Bigg) (\mathbf{d}_j \mathbf{d}_j^H)^{-1} \tag{20}$$

Algorithm 2 Subarray-Based Hybrid WMMSE Algorithm

Set iteration number $n = 0$ and initialize $\mathbf{A}^n$.
repeat
    Update $n = n + 1$.
    Solve (9) for $\mathbf{M}^n$ while $\mathbf{A}^{n-1}$ is fixed.
    Compute $\mathbf{W}^n$ from (15) given $\mathbf{A}^{n-1}$, and $\mathbf{M}^n$.
    Solve (22) for $\{\mathbf{a}_j^n\}$ while $\mathbf{M}^n$ is fixed.
until desired level of convergence

The first order optimality condition of this Lagrangian with respect to $\mathbf{a}_j$ yields (20) in which $\alpha \geq 0$ is chosen via the bisection method such that the transmit power constraint is satisfied. This procedure of alternating optimization continues until a desired level of convergence is achieved. The proposed algorithm converges in terms of objective value since solving a convex problem at each step improves the objective value and the resulting MSE is lower bounded [14]. The optimality of the solution cannot be guaranteed due to the non-convexity of the original problem. Hence, the solution has to be treated as suboptimal unless otherwise proven. Algorithm 1 summarizes the proposed full array-based hybrid WMMSE approach.

B. Subarray-Based Hybrid WMMSE Algorithm

In this algorithm, the digital beamformer is considered to be normalized to identity matrix, i.e., $\mathbf{D} = \mathbf{I}_{N_s}$. Now (13) can be solved by alternating between optimizing the combiner $\mathbf{M}$ and the analog beamformer $\mathbf{A}$. In the first step, the problem is solved with respect to the combiner $\mathbf{M}$. The first order optimality condition of the Lagrangian expression of (13) with respect to $\mathbf{M}$ yields the same MMSE combiner as in (9).

Then, the problem is solved for the analog beamformer $\mathbf{A}$. Using the stream specific MSE expressions, the corresponding Lagrangian expression is given by

$$\mathcal{L} = \sum_{i=1}^{N_s} w_i e_i + \alpha \Big(\sum_{j=1}^{N_a} \mathrm{tr}\,(\mathbf{a}_j \mathbf{a}_j^H) - P \Big). \tag{21}$$

The first order optimality condition of $\mathcal{L}$ with respect to each $\mathbf{a}_j$ yields

$$\mathbf{a}_j = (\mathbf{H}_j^H \mathbf{M}\mathbf{W}\mathbf{M}^H \mathbf{H}_j + \alpha \mathbf{I}_n)^{-1} \mathbf{H}_j^H \mathbf{m}_j w_j \tag{22}$$

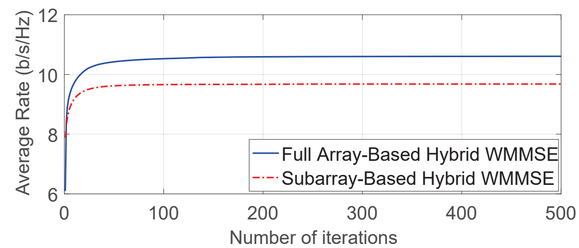
where $\alpha \geq 0$ is chosen via the bisection method while satisfying the transmit power constraint. This alternating optimization procedure is repeated until a desired level of convergence is obtained. The developed subarray-based hybrid WMMSE approach is summarized in Algorithm 2. The convergence and suboptimality properties of Algorithm 1 apply also for Algorithm 2.

V. Simulation Results

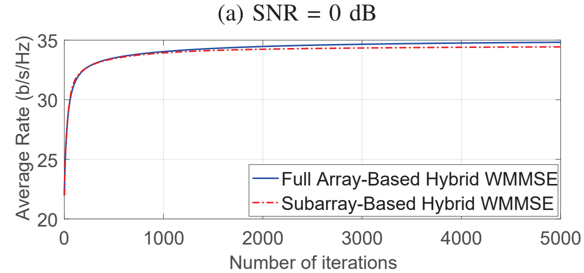
In this section, the rate performance of the proposed HBF algorithms is evaluated and compared to the optimal fully digital and analog beamforming solutions. Moreover, the convergence properties of the algorithms are studied. We use a channel model with geometric uniform linear array (ULA) settings based on Saleh-Valenzuela model [15]. The corresponding MIMO channel matrix is expressed as

$$\mathbf{H} = \sqrt{\frac{N_t N_r}{L}} \sum_{l=1}^{L} \alpha_l \mathbf{g}_r(\phi_r^l) \mathbf{g}_t(\phi_t^l)^H \tag{23}$$

where L is the number of paths between the BS and the user, $\alpha_l \sim \mathcal{CN}(0, 1)$ is the path gain for the lth path, $\phi_r^l \in [0, 2\pi)$ and $\phi_t^l \in [0, 2\pi)$ are the angles of arrival and departure for the lth path, respectively. Moreover, $\mathbf{g}_r(\phi_r^l)$ and $\mathbf{g}_t(\phi_t^l)$ are the receive and transmit antenna array response vectors, respectively. The ULA is considered to have $L = 20$ paths with uniform distribution for both angles of departure and arrival, and the antenna spacing equal to half a wavelength. Moreover, the channel is assumed to be frequency flat with normalized path loss. To reflect the nature of a large scale MIMO system, the number of transmit antennas is set relatively large, i.e., $N_t = 64$. The number of receive antennas is set equal to the number of data streams. The simulation results are averaged over 100 channel realizations.



(a) SNR = 0 dB



(b) SNR = 20 dB

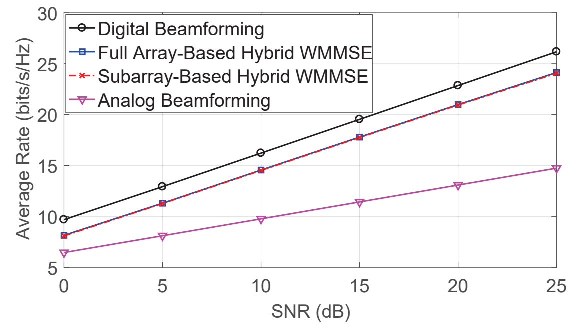
Fig. 3: Convergence behavior with $N_t = 64$, $N_r = 4$, $N_s = 4$.

Figs. 3a, 3b illustrate the convergence behavior of the proposed HBF algorithms for 0 and 20 dB SNRs, respectively. The speed of convergence is relatively fast during the first few iterations, but significantly slows down when the rate gets closer to its limit value. Moreover, the speed gets slower with the increasing SNR. The subarray-based algorithm converges slightly faster than the full array-based method.

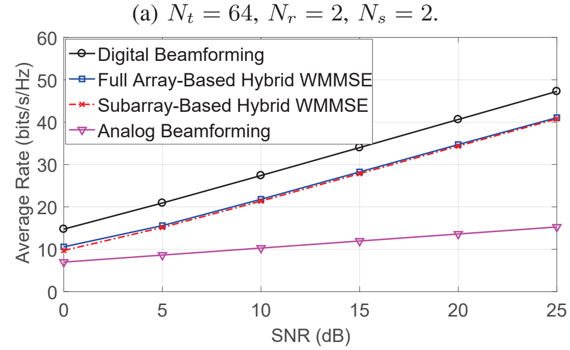
Figs. 4a-4c show rate versus SNR for the developed HBF algorithms in comparison to fully digital and analog beamforming solutions with different numbers of receive antennas and data streams. One can see that partially connected HBF is greatly superior to analog beamforming and slightly inferior to digital precoding. The results imply that properly designed HBF can strike a good balance between spectral efficiency and hardware complexity. Furthermore, the subarray-based hybrid WMMSE algorithm has equal performance to that of the full array-based one when the number of data streams is two. For four streams, the performance is still comparable at medium and high SNRs. In the case of eight-stream MIMO, the full array solution can provide some gain in low SNR regime while the gain becomes marginal as the SNR increases. The provided results are meant as theoretical upper bounds against which more practical HBF methods can be evaluated.

VI. Conclusion

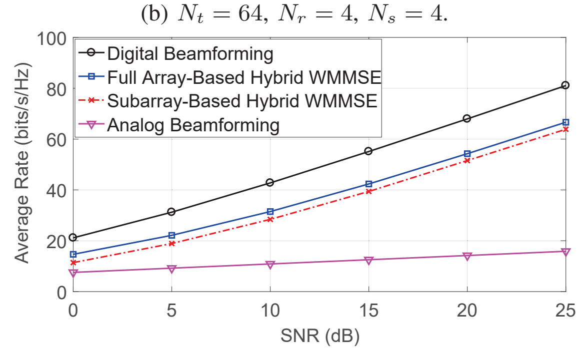
This paper considered partially connected HBF with full array- and subarray-based processing strategies in a SU-MIMO system. We formulated the original HBF rate maximizing problem as a WMMSE problem and proposed two algorithmic solutions based on alternating optimization. The rate performance of the proposed algorithms was evaluated against optimal fully digital and analog beamforming solutions. The results imply that partially connected HBF provides a good trade-off between performance and hardware complexity. Furthermore, the performance of the subarray-based hybrid WMMSE algorithm is comparable to that of the full array-based one at medium and high SNRs and when the number of data streams is four or less. The provided rate maximizing results can be treated as upper bounds for low complexity heuristic approaches. Future work includes studying more practical HBF algorithms in realistic mm-wave massive MIMO channel models.



(a) $N_t = 64$, $N_r = 2$, $N_s = 2$.



(b) $N_t = 64$, $N_r = 4$, $N_s = 4$.



(c) $N_t = 64$, $N_r = 8$, $N_s = 8$.

Fig. 4: Average rate vs. SNR.

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