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Worst Case Age of Information in Wireless Sensor Networks: A Multi-Access Channel

Mohammad Moltafet, Markus Leinonen, and Marian Codreanu

***Abstract*—Freshness of status update packets is essential for enabling a wide range of applications in wireless sensor networks (WSNs). Accordingly, we consider a WSN where sensors communicate status updates to a destination by contending for the channel access based on a carrier sense multiple access (CSMA) method. We analyze the worst case average age of information (AoI) and average peak AoI from the view of one sensor in a system where all the other sensors have a saturated queue. Numerical results illustrate the importance of optimizing the contention window size and the packet arrival rate to maximize the information freshness.**

***Index Terms*– Age of information (AoI), multi-access channel, CSMA/CA, M/G/1 queueing model.**

I. Introduction

In a wide range of Internet of Things (IoT) applications, such as surveillance in smart home systems and drone control, the destination requires the status information of various physical processes collected by multiple sensors. In these applications, timeliness of status information is very critical. The age of information (AoI) was introduced as a destination centric metric that characterizes this timeliness [1]. The AoI for a sensor is defined as the time elapsed since the last received status update packet was generated at the sensor. To evaluate the AoI, the most commonly used metrics are average AoI and peak AoI [2].

Due to the scarcity of radio spectrum and simplicity of devices in wireless sensor networks (WSNs), it is critical to implement an appropriate channel access protocol to efficiently send the status update packets from the sensors to the destination over a shared channel. Carrier sense multiple access with collision avoidance (CSMA/CA) is the most simple and practical contention based access technique in the wireless networks. CSMA/CA is a distributed channel access scheme that allows each sensor to initiate transmissions without any admission whenever a sensor has a data packet to transmit. Two versions of CSMA/CA are employed: I) basic CSMA/CA, and II) CSMA/CA with channel reservation. The basic CSMA/CA is appropriate for the systems with short data packets (e.g., status update systems) and where the hidden node problem is negligible [3].

Many works have evaluated the performance of a CSMA/CA-based system or optimized the system with respect to different criteria (e.g., throughput, delay, etc.). However, there are only a few works that have studied the freshness in a CSMA/CA-based system.

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The authors of [4] investigated the AoI in a CSMA/CA-based vehicular network using simulation. The authors of [5] investigated ALOHA and scheduled based access techniques in WSNs and minimized the average AoI by optimizing the probability of transmission in each node. The most related work to our paper is [6]. The authors of [6] analyzed the AoI in a CSMA/CA-based system using the stochastic hybrid systems technique. To make the analysis tractable, they assumed that there are no collisions in the system by considering that the channel sensing delay is zero and the back-off time is continuous. They considered a system without queueing time where the total capacity of the queuing system is one packet under service. They optimized the average AoI by calibrating the back-off time of each link.

In this paper, we derive the worst case average AoI and average peak AoI of a sensor in a simplified CSMA/CA-based WSN under the first-come first-served (FCFS) policy and infinite queue size. To the best of our knowledge, the existing works have not analytically evaluated the AoI in such a network model. The worst case analysis is carried out by considering that when a sensor contends for the channel to transmit its status update packet, all the other sensors have a packet to transmit. Therefore, we consider a saturated system where the probability of collisions has the highest value. We confine to the commonly used procedure of considering a worst case scenario and a simplified CSMA/CA protocol because a more general case is intractable to analyze [3].

The rest of this paper is organized as follows. Section II presents the system model and the AoI metrics. The worst case average AoI and average peak AoI of the simplified CSMA/CA-based system are derived in Section III. Numerical results are presented in Section IV and conclusions are drawn in Section V.

II. System Model and AoI Metrics

We consider a simplified CSMA/CA-based WSN consisting of M sensors, denoted by $\mathcal{M} = \{1, \ldots, M\}$. Each sensor is assigned to send status update packets of a random process to a destination. We study the AoI of one sensor, m, in a worst case scenario where all the other sensors $m' \in \mathcal{M} \setminus \{m\}$ always have a packet to transmit, i.e., they have saturated queues. In this scenario, the probability of collisions for sensor m has the highest value. We assume that the packet arrival rate of sensor m follows the Poisson process with rate λ, and the server of sensor m works according to the FCFS policy. In the following, a simplified CSMA/CA technique and the AoI metrics are presented.

1) CSMA/CA Mechanism: Here, we briefly present the main concept of the basic CSMA/CA technique, standardized by IEEE 802.11. When sensor m has a packet to transmit, it monitors the shared channel. If the channel is idle for a predetermined period time named distributed interframe space (DIFS), the sensor transmits. Otherwise, if the channel is sensed busy, the sensor persists to monitor the channel until it is found idle for a DIFS period, denoted by T_{DIFS}. The time immediately following an idle DIFS period is *slotted*. At this point, the sensor generates a random number w according to the discrete uniform distribution taking values in $\{1, \dots, C\}$ and sets a back-off counter to the generated number, where C is a fixed contention window size. After choosing the random number, the back-off time counter of the sensor decrements at the beginning of each *slot*. Thus, a slot represents the time interval between two consecutive back-off counter states. When there is no transmission by the other sensors, the back-off counter state decrements after a fixed time interval denoted by T_{F}. When a transmission is detected, the counter is frozen; when the channel is sensed idle for T_{DIFS}, the counter is reactivated. The sensor starts to transmit its data when the counter reaches zero. After transmitting the data, the transmitter senses the channel to detect the acknowledgment (ACK) message from the destination. If the transmitter does not receive ACK within a predetermined time, or it detects a signal of other sensors in the channel, it reschedules the packet transmission according to the random back-off rule. After a successful transmission, if the sensor has a next packet in its buffer, the transmission process is started from the random back-off rule.

2) AoI: A status update packet of each sensor contains the measured value of the monitored process and a time stamp that represents the time when the sample was generated. If at a time instant t, the most recently received status update packet contains the time stamp $U(t)$, AoI is defined as the random process $\Delta(t) = t - U(t)$. In other words, the AoI measures the time elapsed since the last received status update packet was generated at the sensor. The most common metrics of the AoI are average AoI and average peak AoI. The average AoI of a sensor at the sink is the average of $\Delta(t)$, denoted by Δ. The peak AoI of a sensor at the sink is defined as the value of the AoI immediately before receiving a status update packet. The average of the peak AoI is denoted by A.

The considered status update system for sensor m is identical to an M/G/1 queueing model. Let S denote the service time. Then, the average AoI Δ and the average peak AoI A of sensor m is calculated by [7]

$$\Delta = \mathbb{E}[S] + \frac{\lambda \mathbb{E}[S^2]}{2(1-\lambda \mathbb{E}[S])} + \frac{1-\lambda \mathbb{E}[S]}{\lambda L_S(\lambda)}, \tag{1}$$

$$A = \frac{1}{\lambda} + \frac{\lambda \mathbb{E}[S^2]}{2(1-\lambda \mathbb{E}[S])} + \mathbb{E}[S], \tag{2}$$

where $\mathbb{E}[S]$ is the expectation of the service time, $\mathbb{E}[S^2]$ is the second moment of the service time, and $L_S(\lambda) = \mathbb{E}[e^{-\lambda S}]$ is the Laplace transform of the probability distribution function of the service time at the packet arrival rate λ.

In general, the calculation of (1) and (2) requires finding closed-form expressions of the service time parameters $\mathbb{E}[S]$, $\mathbb{E}[S^2]$, and $L_S(\lambda)$ for the basic CSMA/CA-based system. This, however, is intractable due to the intricate nature of the contention mechanism which results in a dependency of the transmissions of the different sensors. In this regard, we use the approximations introduced in [3] as follows. We assume that the probability of a collision in each slot for each sensor has a fixed value which thus disregards the dependencies of the transmission states of other sensors as well as the influence of the number of retransmissions. It is worth noting that when we are evaluating the system from the view point of sensor m, the main analysis intrinsically focuses on the behavior of the system when sensor m has a packet to transmit. On the other hand, the other sensors always have a packet to transmit. Thus, conditioned on the transmission stage of sensor m, each of the M sensors in the network sees $M-1$ sensors that have a packet to transmit. Moreover, we assume that a packet transmission process is started by the random back-off rule and that the ACK message is instantaneous and error-free.

III. Worst Case Average AoI and Average Peak AoI of the Simplified CSMA/CA-Based System

To calculate the average AoI and average peak AoI in (1) and (2), respectively, we next derive the three required quantities $\mathbb{E}[S]$, $\mathbb{E}[S^2]$, and $L_S(\lambda)$. Let K denote a discrete random variable that represents the number of (channel access) attempts sensor m uses to successfully transmit a data packet. Let S_k denote a random variable that represents the service time conditioned on the event that the number of attempts is $K = k$, i.e., the first $k-1$ attempts are failed and the k^{th} attempt is successful. Accordingly, we express S_k as

$$S_k = \textstyle\sum_{j=1}^{k-1} \zeta_j + \xi_k, \tag{3}$$

where ζ_j is a random variable that represents the elapsed time of an unsuccessful transmission of sensor m at the jth attempt and ξ_k is a random variable that represents the elapsed time of a successful transmission of sensor m at the kth attempt.

By using the law of iterated expectations, the expectation $\mathbb{E}[S]$, second moment $\mathbb{E}[S^2]$, and Laplace transform $\mathbb{E}[e^{-\lambda S}]$ are calculated as follows:

$$\begin{aligned}
&\mathbb{E}[S] = \mathbb{E}_K\big[\mathbb{E}[S|K]\big] = \textstyle\sum_{k=1}^{\infty} \mathbb{E}[S_k]\Pr(K=k),\\
&\mathbb{E}[S^2] = \mathbb{E}_K\big[\mathbb{E}[S^2|K]\big] = \textstyle\sum_{k=1}^{\infty} \mathbb{E}[S_k^2]\Pr(K=k),\\
&\mathbb{E}[e^{-\lambda S}] = \mathbb{E}_K\big[\mathbb{E}[e^{-\lambda S}|K]\big] = \textstyle\sum_{k=1}^{\infty} \mathbb{E}\big[e^{-\lambda S_k}\big]\Pr(K=k),
\end{aligned} \tag{4}$$

where $\Pr(K=k)$ is the probability of the event that $k-1$ attempts are failed and the k^{th} attempt is successful. We can see from (4) that to calculate the expectations $\mathbb{E}[S]$, $\mathbb{E}[S^2]$, and $\mathbb{E}[e^{-\lambda S}]$ we need to calculate the quantities $\mathbb{E}[S_k]$, $\mathbb{E}[S_k^2]$, $\mathbb{E}[e^{-\lambda S_k}]$, and $\Pr(K=k)$. These are derived in the following.

1) Calculation of $\mathbb{E}[S_k]$: By using (3), $\mathbb{E}[S_k]$ in (4) is written as follows:

$$\mathbb{E}[S_k] = \textstyle\sum_{j=1}^{k-1} \mathbb{E}[\zeta_j] + \mathbb{E}[\xi_k]. \tag{5}$$

First, we derive $\mathbb{E}[\xi_k]$. Let a discrete random variable W represent the generated random number by sensor m in the back-off rule. Let $\xi_{j,w}$ denote the elapsed time of a successful transmission of sensor m at the jth attempt conditioned on the event that the generated number is $W = w$. Then, by using the law of iterated expectations, $\mathbb{E}[\xi_j]$ can be calculated as follows:

$$\mathbb{E}[\xi_j] = \mathbb{E}_W\big[\mathbb{E}[\xi_j|W]\big] = \textstyle\sum_{w=1}^{C} \mathbb{E}[\xi_{j,w}]\Pr(W=w) \tag{6}$$

$$\stackrel{(a)}{=} \sum_{w=1}^{C} \frac{\mathbb{E}[\xi_{j,w}]}{C},$$

where equality (a) follows because the random number W is selected according to the uniform distribution, i.e., $\Pr(W = w) = 1/C$.

By the definition of a successful transmission, $\xi_{j,w}$ in (6) is equal to the summation of the elapsed time until the back-off time counter reaches zero and the required time to transmit a packet, i.e.,

$$\xi_{j,w} = \sum_{i=1}^{w} T_{j,i} + T_{\mathrm{P}}, \tag{7}$$

where $T_{j,i}$ is a random variable that represents the time interval between two consecutive back-off counter states i and $i-1$ at the jth attempt and T_{P} is the required time to transmit a data packet (which is determined according to the channel rate, data packet size etc.). By substituting (7) in (6), we have

$$\mathbb{E}[\xi_j] = \frac{1}{C} \sum_{w=1}^{C} \big(\sum_{i=1}^{w} \mathbb{E}[T_{j,i}] + T_{\mathrm{P}}\big). \tag{8}$$

Similarly as for $\mathbb{E}[\xi_j]$, we derive $\mathbb{E}[\zeta_j]$ by introducing a random variable $\zeta_{j,w}$ to describe the elapsed time of an unsuccessful transmission of sensor m at the jth attempt conditioned on the event that the generated number in the back-off rule is $W = w$. Thus, $\mathbb{E}[\zeta_j]$ can be calculated as follows:

$$\mathbb{E}[\zeta_j] = \mathbb{E}_W\big[\mathbb{E}[\zeta_j|W]\big] = \sum_{w=1}^{C} \mathbb{E}[\zeta_{j,w}]\Pr(W = w) \tag{9}$$
$$= \sum_{w=1}^{C} \frac{\mathbb{E}[\zeta_{j,w}]}{C}.$$

The random variable $\zeta_{j,w}$ is equal to the summation of the elapsed time until the back-off time counter reaches zero and the required time to transmit a packet, i.e.,

$$\zeta_{j,w} = \sum_{i=1}^{w} T_{j,i} + T_{\mathrm{P}}. \tag{10}$$

By substituting (10) in (9), we have

$$\mathbb{E}[\zeta_j] = \frac{1}{C} \sum_{w=1}^{C} \big(\sum_{i=1}^{w} \mathbb{E}[T_{j,i}] + T_{\mathrm{P}}\big). \tag{11}$$

To calculate $\mathbb{E}[\xi_j]$ in (8) and $\mathbb{E}[\zeta_j]$ in (11), we need to calculate $\mathbb{E}[T_{j,i}]$. The random variable $T_{j,i}$ can be defined based on two events: 1) When there is no transmission by the other sensors $m' \in \mathcal{M} \setminus \{m\}$ between two back-off counter states i and $i-1$, we have $T_{j,i} = T_{\mathrm{F}}$, where T_{F} is the maximum duration that the sensor persists to sense the idle channel before decrementing the back-off counter; 2) When there is at least one transmission (successful or unsuccessful) of the other sensors $m' \in \mathcal{M} \setminus \{m\}$ between two consecutive back-off counter states i and $i-1$, the time between states i and $i-1$ is equal to the summation of the required time to transmit a packet T_{P}, the DIFS period T_{DIFS}, and a fraction of the slot size T_{F} (i.e., the time interval between the time instant that the back-off counter state is decremented to i and the time instant that sensor m detects a transmission of other sensors). However, similarly as in the analysis in [3], we neglect this fraction of the slot size in computing $T_{j,i}$, and thus we have $T_{j,i} = T_{\mathrm{P}} + T_{\mathrm{DIFS}}$. It is worth to note that in a CSMA/CA based system, T_F is significantly smaller than the time period $T_{\mathrm{P}} + T_{\mathrm{DIFS}}$. For example, with the considered parameters in our numerical results in Section IV, T_F is less than 2% of $T_{\mathrm{P}} + T_{\mathrm{DIFS}}$. Let $P_{j,i}^{\mathrm{tr}}$ denote the probability of having at least one transmission by the other sensors between two consecutive back-off counter states i and $i-1$ at jth attempt. Thus, $\mathbb{E}[T_{j,i}]$ is given by

$$\mathbb{E}[T_{j,i}] = P_{j,i}^{\mathrm{tr}}(T_{\mathrm{P}} + T_{\mathrm{DIFS}}) + (1 - P_{j,i}^{\mathrm{tr}})T_{\mathrm{F}}. \tag{12}$$

Due to the considered assumptions, the probability of having at least one transmission by the other sensors between each two consecutive back-off counter states at each attempt has a fixed value and we have [3]

$$P_{j,i}^{\mathrm{tr}} = P^{\mathrm{tr}} = 1 - \left(\frac{C-1}{C+1}\right)^{M-1}, \forall i, j. \tag{13}$$

Therefore, (12) becomes

$$\mathbb{E}[T_{j,i}] = \mathbb{E}[T] = (1 - P^{\mathrm{tr}})T_{\mathrm{F}} + P^{\mathrm{tr}}(T_{\mathrm{P}} + T_{\mathrm{DIFS}}), \forall i, j. \tag{14}$$

Considering (14) and the expressions for $\mathbb{E}[\xi_j]$ and $\mathbb{E}[\zeta_j]$ in (8) and (11), respectively, we have:

$$\mathbb{E}[\xi_j] = \mathbb{E}[\zeta_{j'}] \triangleq \bar{\xi}_1, \ \ \forall j, j', \tag{15}$$

where, by substituting (14) in (8), $\bar{\xi}_1$ is calculated as

$$\bar{\xi}_1 = \big((C+1)\mathbb{E}[T]\big)/2 + T_{\mathrm{P}}. \tag{16}$$

Finally, by using (15) in (5), we have

$$\mathbb{E}[S_k] = k\bar{\xi}_1. \tag{17}$$

2) Calculation of $\mathbb{E}[S_k^2]$: By using (3), $\mathbb{E}[S_k^2]$ in (4) is calculated as follows:

$$\mathbb{E}[S_k^2] = \mathbb{E}\big[\big(\sum_{j=1}^{k-1} \zeta_j + \xi_k\big)^2\big] = \mathbb{E}\big[\sum_{j=1}^{k-1} \zeta_j^2\big] \tag{18}$$
$$+ 2\mathbb{E}\big[\sum_{j=1}^{k-1} \sum_{j'=1, j' \neq j}^{k-1} \zeta_j \zeta_{j'}\big] + \mathbb{E}\big[\xi_k^2\big] + 2\mathbb{E}\big[\xi_k \sum_{j=1}^{k-1} \zeta_j\big].$$

Since the elapsed time of each channel access attempt is independent of the elapsed time of the other attempts, we have

$$\mathbb{E}[\zeta_j \zeta_{j'}] = \mathbb{E}[\zeta_j]\mathbb{E}[\zeta_{j'}], \forall j, j', \ \ j \neq j'$$
$$\mathbb{E}[\zeta_j \xi_{j'}] = \mathbb{E}[\zeta_j]\mathbb{E}[\xi_{j'}], \forall j, j', \ \ j \neq j'. \tag{19}$$

By means of (19), $\mathbb{E}[S_k^2]$ in (18) can be presented as a function of $\bar{\xi}_1$ in (16), $\mathbb{E}[\xi_k^2]$, and $\mathbb{E}[\zeta_j^2], j = \{1, \ldots, k-1\}$. By following steps similar to (6)-(15) for $\mathbb{E}[\xi_j^2]$ and $\mathbb{E}[\zeta_j^2]$, it is easy to show that

$$\mathbb{E}[\xi_j^2] = \mathbb{E}[\zeta_{j'}^2] \triangleq \bar{\xi}_2, \ \ \forall j, j', \tag{20}$$

where $\bar{\xi}_2$ is calculated using the steps similar as for $\bar{\xi}_1$, resulting in

$$\bar{\xi}_2 = \frac{1}{C} \sum_{w=1}^{C} \big(2w\mathbb{E}[T]T_{\mathrm{P}} + T_{\mathrm{P}}^2 + w\mathbb{E}[T^2] + w(w-1)\mathbb{E}[T]^2\big)$$
$$\stackrel{(a)}{=} T_{\mathrm{P}}^2 + (C+1)\left[\frac{(2\mathbb{E}[T]T_{\mathrm{P}} + \mathbb{E}[T^2] - \mathbb{E}[T]^2)}{2} \right. \tag{21}$$
$$\left. + \frac{(2C+1)\mathbb{E}[T]^2}{6}\right],$$

where equality (a) follows from the following feature of the finite series [8, Sect. 1.5]

$$\sum_{u=1}^{U} u^2 = \big(U(U+1)(2U+1)\big)/6,$$

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$\mathbb{E}[T]$ is calculated by (14), and $\mathbb{E}[T^2]$ is given by

$$\mathbb{E}[T^2] = (1 - P^{\text{tr}})T_{\text{F}}^2 + P^{\text{tr}}(T_{\text{P}} + T_{\text{DIFS}})^2. \tag{22}$$

Finally, by applying (15), (19), and (20), (18) is written by

$$\mathbb{E}[S_k^2] = k\bar{\xi}_2 + k(k-1)\bar{\xi}_1^2. \tag{23}$$

3) Calculation of $\mathbb{E}[e^{-\lambda S_k}]$*:* By substituting (3) in $\mathbb{E}[e^{-\lambda S_k}]$, we have

$$\mathbb{E}[e^{-\lambda S_k}] = \mathbb{E}\big[\textstyle\prod_{j=1}^{k-1} e^{-\lambda\zeta_j} e^{-\lambda\xi_k}\big]. \tag{24}$$

Due to the fact that elapsed time of different attempts are independent of each other, we have

$$\begin{aligned} &\mathbb{E}[e^{-\lambda\zeta_j} e^{-\lambda\zeta_{j'}}] = \mathbb{E}[e^{-\lambda\zeta_j}]\mathbb{E}[e^{-\lambda\zeta_{j'}}], \forall j, j', \;\; j \neq j' \\ &\mathbb{E}[e^{-\lambda\zeta_j} e^{-\lambda\xi_{j'}}] = \mathbb{E}[e^{-\lambda\zeta_j}]\mathbb{E}[e^{-\lambda\xi_{j'}}], \forall j, j', \;\; j \neq j'. \end{aligned} \tag{25}$$

By applying (25), (24) is written as follows:

$$\mathbb{E}[e^{-\lambda S_k}] = \textstyle\prod_{j=1}^{k-1} \mathbb{E}\big[e^{-\lambda\zeta_j}\big]\mathbb{E}\big[e^{-\lambda\xi_k}\big]. \tag{26}$$

By following steps similar to (6)-(15) for $\mathbb{E}[e^{-\lambda\zeta_j}]$ and $\mathbb{E}[e^{-\lambda\xi_j}]$, it can be shown that

$$\mathbb{E}[e^{-\lambda\zeta_j}] = \mathbb{E}[e^{-\lambda\xi_{j'}}] \triangleq \bar{\xi}_3, \;\; \forall j, j', \tag{27}$$

where $\bar{\xi}_3$ is given as

$$\begin{aligned} \bar{\xi}_3 &= \frac{1}{C}\textstyle\sum_{w=1}^{C} \left(\mathbb{E}[e^{-\lambda T}]^w e^{-\lambda T_{\text{P}}}\right) \\ &\overset{(a)}{=} \frac{e^{-\lambda T_{\text{P}}}\mathbb{E}[e^{-\lambda T}](1 - \mathbb{E}[e^{-\lambda T}]^C)}{C(1 - \mathbb{E}[e^{-\lambda T}])}, \end{aligned} \tag{28}$$

where equality (a) follows from the following feature of the finite series [8, Sect. 1.5], $\sum_{u=1}^{U} \alpha^u = \dfrac{\alpha(1-\alpha^U)}{1-\alpha}$, and $\mathbb{E}[e^{-\lambda T}]$ is given by

$$\mathbb{E}[e^{-\lambda T}] = (1 - P^{\text{tr}})e^{-\lambda T_{\text{F}}} + P^{\text{tr}} e^{-\lambda(T_{\text{P}} + T_{\text{DIFS}})}. \tag{29}$$

Finally, using (27), $\mathbb{E}[e^{-\lambda S_k}]$ in (26) is written as

$$\mathbb{E}[e^{-\lambda S_k}] = \bar{\xi}_3^k. \tag{30}$$

4) Calculation of $\Pr(K = k)$*:* Due to the considered assumptions, the probability of having a successful transmission in each attempt has a fixed value which is denoted by P_{S}. Therefore, $\Pr(K = k)$ is given by

$$\Pr(K = k) = P_{\text{S}}(1 - P_{\text{S}})^{k-1}, \tag{31}$$

where P_{S} is given by [3]

$$P_{\text{S}} = \left(\frac{C-1}{C+1}\right)^{M-1}. \tag{32}$$

5) Final expressions for $\mathbb{E}[S]$, $\mathbb{E}[S^2]$, *and* $L_S(\lambda)$ *in* (4)*:* Substituting (17), (23), (30), and (31) in (4), we have

$$\begin{aligned} &\mathbb{E}[S] = \textstyle\sum_{k=1}^{\infty} k\bar{\xi}_1 P_{\text{S}}(1 - P_{\text{S}})^{k-1}, \\ &\mathbb{E}[S^2] = \textstyle\sum_{k=1}^{\infty} \left(k\bar{\xi}_2 + k(k-1)\bar{\xi}_1^2\right)P_{\text{S}}(1 - P_{\text{S}})^{k-1}, \\ &\mathbb{E}[e^{-\lambda S}] = \textstyle\sum_{k=1}^{\infty} \bar{\xi}_3^k P_{\text{S}}(1 - P_{\text{S}})^{k-1}. \end{aligned} \tag{33}$$

According to the feature of the series, for each $0 \leq \alpha < 1$, we have [8, Sect. 8.6]

$$\textstyle\sum_{u=1}^{\infty} u\alpha^u = \dfrac{\alpha}{(1-\alpha)^2}, \;\; \textstyle\sum_{u=1}^{\infty} u^2\alpha^u = \dfrac{\alpha(1+\alpha)}{(1-\alpha)^3}. \tag{34}$$

Thus, by applying (34) in (33), $\mathbb{E}[S]$, $\mathbb{E}[S^2]$, and $\mathbb{E}[e^{-\lambda S}]$ are calculated as follows:

$$\begin{aligned} &\mathbb{E}[S] = \frac{\bar{\xi}_1}{P_{\text{S}}}, \;\; \mathbb{E}[S^2] = \frac{\bar{\xi}_2}{P_{\text{S}}} + \frac{\bar{\xi}_1^2(2 - 2P_{\text{S}})}{P_{\text{S}}^2}, \\ &L_S(\lambda) = \begin{cases} \dfrac{\bar{\xi}_3 P_{\text{S}}}{1 - \bar{\xi}_3 + \bar{\xi}_3 P_{\text{S}}}, & \bar{\xi}_3(1 - P_{\text{S}}) < 1, \\ \infty, & \text{Otherwise}, \end{cases} \end{aligned} \tag{35}$$

with $\bar{\xi}_1$, $\bar{\xi}_2$, and $\bar{\xi}_3$ given in (16), (21), and (28), respectively. As the outcome, the expressions in (35) can be used to calculate the average AoI in (1) and the average peak AoI in (2) in the considered model.

IV. Numerical Results

In this section, we present numerical results to show the behavior of the AoI for different system parameters. We set $T_{\text{DIFS}} = 128\,\mu s$, $T_{\text{F}} = 50\,\mu s$, channel bit rate 1 Mbit/s, and packet size 300 Bytes.

Figs. 1 and 2 depict the average AoI and average peak AoI of sensor m as a function of the packet arrival rate λ for different number of sensors with contention window sizes $C = \{100, 800\}$, respectively. When the number of sensors M increases, the average AoI and average peak AoI dramatically increase because the probability of collisions in the system increases. It is worth noting that, when M increases, the values of the packet arrival rate λ that minimize the average AoI and average peak AoI both decrease. In addition, the curvatures demonstrate that the range of values of λ that result in near-optimal AoI become narrower for the increasing values of M. This emphasizes the importance of implementing an optimal generation policy of status update packets in WSNs with a shared-access channel.

Fig. 3 illustrates the average AoI of sensor m as a function of λ for different contention window sizes with a fixed number of sensors $M = 100$. According to this figure, naively increasing (or decreasing) the contention window size does not minimize the average AoI and average peak AoI. Namely, $C = 1000$ leads to a smaller average AoI than both $C = 500$ and $C = 1500$. However, for all larger contention window sizes $C = \{500, 1000, 1500\}$, the AoI is not very sensitive to the packet arrival rate λ in the sense that a wide range of values of λ result in relatively low values of the average AoI.

Fig 4 illustrates the optimal value of the contention window size C as a function of the number of sensors M and the arrival rate λ. According to this figure, when the number of sensors increases, the optimal contention window size C increases. This is because when C increases, the probability of a collision decreases. In other words, increasing the size of the contention window mitigates the effect of an increased number of sensors on the probability of a collision. However, C can not be set by an arbitrary large number because it results in a high value of the average AoI. In addition, the curve shows that for the small number of sensors, when the packet arrival rate λ increases, the optimal C increases smoothly. Moreover, the figure illustrates that for a large number of sensors, when λ increases, the optimal C first increases and then decreases. This represents the trade-off between the effect of the probability of a collision

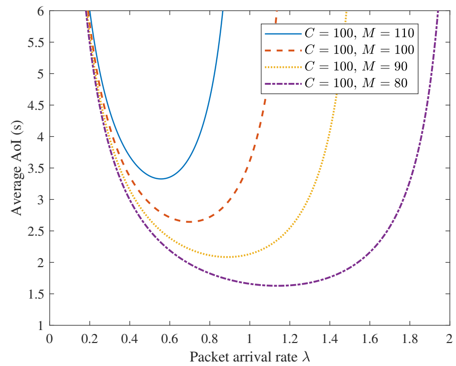

Fig. 1: The average AoI of sensor m as a function of λ for different number of sensors with a fixed contention window size $C = 100$.

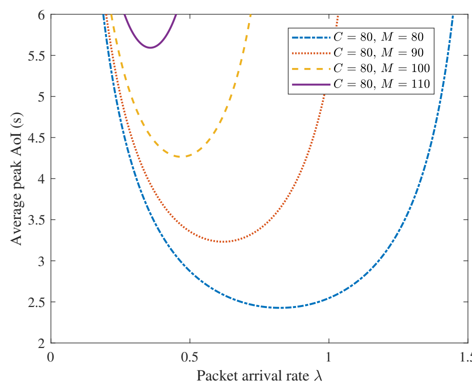

Fig. 2: The average peak AoI of sensor m as a function of λ for different number of sensors with $C = 80$.

and the delay imposed by the contention window size on the average AoI.

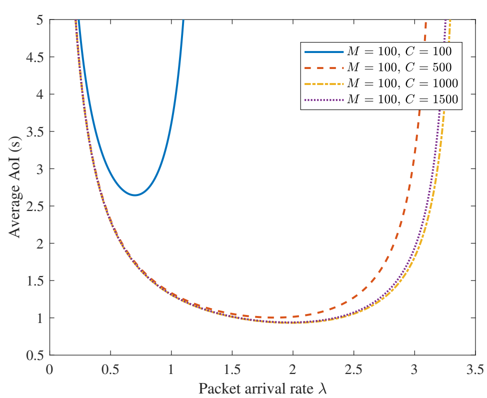

Fig. 3: The average AoI of sensor m as a function of λ for different contention window sizes with a fixed number of sensors $M = 100$.

V. Conclusion

In this paper, we analytically evaluated the worst case average AoI and average peak AoI of a simplified CSMA/CA-based system. The worst case analysis was carried out by considering a scenario in which the probability of collisions for one considered sensor has the highest value. According to the numerical results, the number of contending sensors significantly affects the AoI due to network congestion. The experiments illustrated that optimizing the contention window size and the packet arrival rate can significantly improve the freshness of status updates in the considered system. The interesting future work would be to address a more realistic setup, where all sensors have bursty arrivals instead of saturated queues. However, due to the complex interactions of queues, an exact analysis is most likely analytically intractable even in a simplified homogeneous scenario, where all sensors have equal packet arrival rates and thus, one may need to resort on numerical simulations.

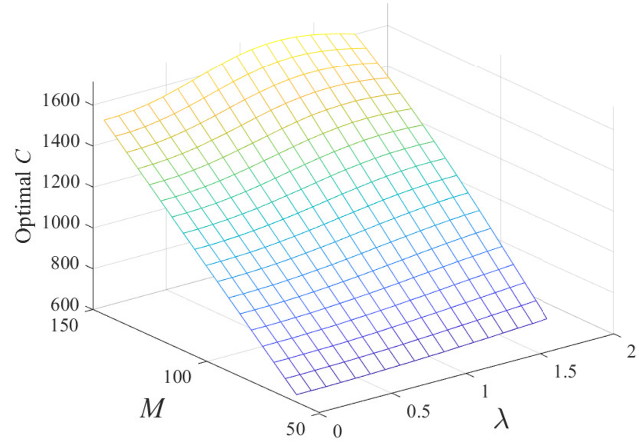

Fig. 4: The optimal value of the contention window size C as a function of the number of sensors M and arrival rate λ.

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