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Research Article

Power Control via Stackelberg Game for Small-Cell Networks

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In this paper, power control in the uplink for two-tier small-cell networks is investigated. We formulate the power control problem as a Stackelberg game, where the macrocell user equipment (MUE) acts as the leader and the small-cell user equipment (SUE) acts as the follower. To reduce the cross-tier and cotier interferences and the power consumption of both the MUE and SUE, we propose optimizing not only the transmit rate but also the transmit power. The corresponding optimization problems are solved through a two-layer iteration. In the inner iteration, the SUE items (SUEs) compete with each other, and their optimal transmit powers are obtained through iterative computations. In the outer iteration, the optimal transmit power of the MUE is obtained in a closed form based on the transmit powers of the SUEs through proper mathematical manipulations. We prove the convergence of the proposed power control scheme, and we also theoretically show the existence and uniqueness of the Stackelberg equilibrium (SE) in the formulated Stackelberg game. The simulation results show that the proposed power control scheme provides considerable improvements, particularly for the MUE.

1. Introduction

With the rapid development of the global communications industry, the problem of high energy consumption of communications systems is becoming increasingly more serious, and determining how to effectively improve the energy efficiency of the entire network is becoming increasingly more urgent. The introduction of small cells can greatly reduce the energy consumption of the entire network. Furthermore, because small cells have small-cell radii with small base stations (SBSs) deployed closer to users and because short-distance transmissions have smaller path loss and fading compared to long-distance transmissions, the throughput of the entire network can be increased. Therefore, the energy efficiency of the entire two-tier small-cell network, which is composed of macrocells and a large number of small cells, can be greatly improved [1]. To improve the spectral efficiency, small cells can share the spectrum with macrocells; however, the cotier and cross-tier interferences will seriously degrade the system performance due to the sharing of the spectrum. In this regard, proper power control in small-cell networks is required to reduce the interference and the power consumption.

Power control is an important research topic that has been widely investigated in the literature [2–5]. In two-tier small-cell networks, small cells can be deployed randomly and freely, and game theory has increasingly been used to achieve distributed power control [6, 7]. In [8], the interference dynamics caused by time-varying environment was considered and a robust mean field game was proposed to control the transmit power of SBSs. In [9–11], it was shown that a Stackelberg game can provide a suitable framework for modeling the competition in two-tier networks. Specifically, a power control problem was formulated to maximize energy efficiency with minimal information exchange in [9]. In [10], both uniform and nonuniform pricing schemes were proposed to obtain the optimal resource allocation with a tolerable interference power constraint. In [11], a network interference controller was proposed to minimize the sum interference by pricing the power consumption. However, most of the existing literature only addressed power control in the downlink and ignored power control of both small-cell user equipment (SUE) and macrocell user equipment (MUE) in the uplink. (Note that the UE served by an SBS is called SUE, and the UE served by an MBS is called MUE.) Moreover, because most of the existing literature addressed power control through a price-based Stackelberg game, they can determine the optimal price and power control for only one type of device. Therefore, if we formulate the power control problem through a Stackelberg game without pricing, the optimal power control for the two types of UEs in the considered two-tier small-cell networks can be determined simultaneously.

Motivated by the aforementioned discussions, we develop a power control scheme taking both SUEs and MUE into account. First, the power control problem of the considered two-tier small-cell networks is mathematically formulated as a Stackelberg game that consists of one leader and multiple followers. Second, the optimization problem is solved through a two-layer iteration. In the inner iteration, the followers compete with each other, and their optimal transmit powers are obtained through iterative computations. In the outer iteration, the optimal transmit power of the leader is calculated based on the transmit powers of the followers. Then, we theoretically show the convergence of the proposed scheme and the existence and uniqueness of the Stackelberg equilibrium (SE) in the Stackelberg game. Finally, our proposed power control scheme is verified through simulations, showing that it greatly improves the performance of the MUE.

The remainder of this paper is organized as follows. In Section 2, the system model of the considered two-tier small-cell networks is presented. In Section 3, the proposed power control scheme via a Stackelberg game is developed. The simulation results are presented in Section 4. Final conclusions are drawn in Section 5.

2. System Model

Consider the two-tier small-cell network shown in Figure 1, which consists of one macrocell and K small cells. Assume that the macro base station (MBS) and SBSs share the same spectrum and that only one UE communicates with each BS at any time. In the uplink, each SBS will experience interference from the MUE and its nearby SUEs, and the MBS will experience interference from its nearby SUEs. Let P_0 denote the transmit power of the MUE served by the MBS, P_k denote the transmit power of the kth SUE, and $\boldsymbol{p} = [P_1, P_2, \ldots, P_k, \ldots, P_K]^T$ denote the transmit power vector of the considered K SUEs. Then, the transmit rate of the MUE served by the MBS can be expressed as follows:

$$R_0(P_0, \boldsymbol{p}) = \ln\left(1 + \frac{H_{00}P_0}{N_0 + \sum_{k=1}^{K} H_{0k}P_k}\right), \quad (1)$$

where H_{00} denotes the channel gain from the MUE to its corresponding MBS, H_{0k} is the interference channel gain from the kth SUE to the MBS, and N_0 is the noise power.

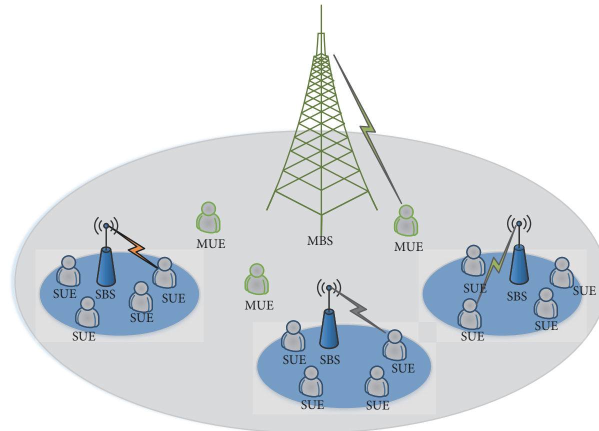

Figure 1: The schematic of the considered two-tier small-cell network.

The transmit rate of the SUE served by the kth SBS can be expressed as follows:

$$\begin{aligned} &R_k(P_k, \boldsymbol{p}_{-k}, P_0) \\ &\quad = \ln\left(1 + \frac{H_{kk}P_k}{N_0 + H_{k0}P_0 + \sum_{k'\neq k, k'=1}^{K} H_{kk'}P_{k'}}\right), \end{aligned} \quad (2)$$

where $\boldsymbol{p}_{-k}$ denotes the transmit power vector of the $K-1$ other SUEs and $\boldsymbol{p}_{-k} = [P_1, P_2, \ldots, P_{k-1}, P_{k+1}, \ldots, P_K]^T$, H_{kk} is the channel gain from the kth SUE to its corresponding SBS, H_{k0} is the interference channel gain from the MUE to the kth SBS, and $H_{kk'}$ is the interference channel gain from the k'th SUE to the kth SBS.

The design objective of this paper is to develop a power control scheme that can increase the transmit rate with reduced cotier and cross-tier interferences and power consumption. Moreover, this paper aims to achieve the above design objective for two-tier small-cell networks where there are two types of UEs, i.e., MUE and SUE, and two different cell types, i.e., macrocell and small cell.

3. The Proposed Power Control Scheme via Stackelberg Game

In this section, we propose a power control scheme for two-tier small-cell networks based on a Stackelberg game, which has one leader and multiple followers. In the formulated Stackelberg game, the MUE, acting as the leader, is supposed to make its own decision and maximize its utility with the best responses of the followers, and the SUEs acting as the followers will respond to the leader's action and maximize their utilities through a subgame [12–14]. Note that the transmit power of the MUE or SUE is controlled by its corresponding MBS or SBS and the MBS can control its corresponding SBSs in the considered two-tier small-cell network. When the transmit power of the MUE has been determined by the MBS, this transmit power information will be sent from the MBS to its corresponding SBSs. Therefore, the MUE controlled by the MBS acts as the leader, and the SUEs controlled by its corresponding SBSs act as the followers.

3.1. Stackelberg Game Formulation. From (1) and (2), we find that the transmit rate of the MUE can be improved by increasing the transmit power of the MUE but at the cost of increased cross-tier interference to the SUEs. Likewise, the transmit rate of the SUEs can be improved by increasing the transmit power of the corresponding SUE but at the cost of increased cross-tier interference to the MUE and increased cotier interference to the other $K-1$ SUEs. To reduce the cross-tier and cotier interferences and the power consumption of both the MUE and SUEs, we propose optimizing not only the transmit rate but also the power consumption. First, the leader MUE moves and determines its transmit power. Subsequently, the follower SUEs move and update their power control strategies to maximize their individual utilities based on the MUE's transmit power.

We define the utility function of the MUE as follows:

$$U_0(P_0, \boldsymbol{p}) = R_0(P_0, \boldsymbol{p}) - \lambda_0 P_0, \tag{3}$$

where λ_0 denotes the coefficient characterizing the influence of per unit transmission power for MUE [15]. Then, the optimization problem of the MUE can be expressed as follows:

$$\begin{aligned} \max_{P_0} \quad & U_0(P_0, \boldsymbol{p}), \\ \text{s.t.} \quad & 0 \le P_0 \le P_T, \end{aligned} \tag{4}$$

where P_T denotes the maximum transmit power of the MUE or SUEs.

We define the utility function of the kth SUE as follows:

$$U_k(P_k, \boldsymbol{p}_{-k}, P_0) = R_k(P_k, \boldsymbol{p}_{-k}, P_0) - \lambda_k P_k, \tag{5}$$

where λ_k denotes the coefficient characterizing the influence of per unit transmission power for SUE. Then, the optimization problem of the kth SUE can be expressed as follows:

$$\begin{aligned} \max_{P_k} \quad & U_k(P_k, \boldsymbol{p}_{-k}, P_0), \\ \text{s.t.} \quad & 0 \le P_k \le P_T, \quad \forall k \in \{1, 2, \ldots, K\}. \end{aligned} \tag{6}$$

The optimization problems in (4) and (6) lead to a Stackelberg game. In this game, the objective is to find the SE point from which neither the leader nor the followers have incentives to deviate. Just similar to the definition in [10], we define the SE as follows.

Definition 1. Let P_0^* and P_k^* denote the two solutions for the optimization problems in (4) and (6), respectively. Let $\boldsymbol{p}^* = [P_1^*, P_2^*, \ldots, P_k^*, \ldots, P_K^*]^T$ and $\boldsymbol{p}_{-k}^* = [P_1^*, P_2^*, \ldots, P_{k-1}^*, P_{k+1}^*, \ldots, P_K^*]^T$. Then, $(P_0^*, \boldsymbol{p}^*)$ is an SE point for the proposed Stackelberg game if the following conditions are satisfied:

$$U_0(P_0^*, \boldsymbol{p}^*) \ge U_0(P_0, \boldsymbol{p}^*), \tag{7}$$

$$U_k(P_k^*, \boldsymbol{p}_{-k}^*, P_0^*) \ge U_k(P_k, \boldsymbol{p}_{-k}^*, P_0^*). \tag{8}$$

Generally, the SE for a Stackelberg game can be obtained by finding its subgame perfect Nash equilibrium (NE) [10, 16]. In our proposed Stackelberg game, it can be readily seen that the SUEs compete in a noncooperative fashion. Therefore, a noncooperative power control subgame is formulated, where the corresponding NE is defined as the operating point at which no player can improve utility by changing its strategy unilaterally [10].

To obtain the SE of the proposed Stackelberg game, we propose exploiting the backward induction method [17] to solve the above optimization problems. Generally, the followers' best responses can be obtained with the fixed value given by the leader, and then the optimal strategy of the leader can be achieved according to the followers' best responses. Correspondingly, we can first solve the followers' optimization problem in (6). Then, by using the obtained solution, we can solve the leader's optimization problem in (4).

3.2. The Optimal Solution of the Followers' Optimization Problem. We have the following theorem for the optimal solution of the optimization problem in (6) for the followers.

Theorem 2. *Given the transmit power of the MUE, the optimization problem in (6) has a globally optimal solution, as follows:*

$$\widetilde{P}_k^* = \begin{cases} P_T, & P_k^{tmp} > P_T, \\ P_k^{tmp}, & 0 < P_k^{tmp} \le P_T, \\ 0, & P_k^{tmp} \le 0, \end{cases} \tag{9}$$

or

$$\widetilde{P}_k^* = P_T - \left[P_T - \left(P_k^{tmp}\right)^+\right]^+, \tag{10}$$

where $(\cdot)^+ \triangleq \max(\cdot, 0)$,

$$P_k^{tmp} = \frac{1}{\lambda_k} - \frac{N_0 + H_{k0}P_0 + \sum_{k' \ne k, k'=1}^{K} H_{kk'} P_{k'}}{H_{kk}}, \tag{11}$$

and P_k^{tmp} *denotes the temporary value of the optimal transmit power of the kth SUE.*

Proof. As shown, the utility function $U_k(P_k, \boldsymbol{p}_{-k}, P_0)$ is strictly concave. Furthermore, it can be verified that the SUEs' strategy space is a nonempty and close-bounded convex set in Euclidean space. Correspondingly, the optimization problem in (6) can readily be proven to be convex; thus, it has a globally optimal solution. By setting the first-order derivative of $U_k(P_k, \boldsymbol{p}_{-k}, P_0)$ with respect to P_k to zero, P_k^{tmp} can readily be calculated as shown in (11). By considering the constraint $0 \le P_k \le P_T$, the optimal solution of the optimization problem in (6) can readily be obtained as shown in (9) or (10). This completes the proof. □

3.3. The Optimal Solution of the Leader's Optimization Problem. After some mathematical manipulations, we can obtain the refined constraint for P_0 according to (9) as follows:

$$P_0^{\min} \le P_0 \le P_0^{\max}, \tag{12}$$

where

$$P_0^{\min} = \max_{k\in\{1,2,\dots,K\}} P_{0,k}^{\min}, \tag{13}$$

$$P_0^{\max} = \min_{k\in\{1,2,\dots,K\}} P_{0,k}^{\max}, \tag{14}$$

and $P_{0,k}^{\min}$ and $P_{0,k}^{\max}$ are shown in (15) and (16), respectively.

$$P_{0,k}^{\min} = \begin{cases} 0, & P_k^{\text{tmp}} > P_T, \\ \max\left\{0, \dfrac{(1/\lambda_k - P_T) H_{kk} - N_0 - \sum_{k'\neq k,k'=1}^{K} H_{kk'}\widetilde{P}_{k'}^*}{H_{k0}}\right\}, & 0 < P_k^{\text{tmp}} \le P_T, \\ \max\left\{0, \dfrac{(1/\lambda_k) H_{kk} - N_0 - \sum_{k'\neq k,k'=1}^{K} H_{kk'}\widetilde{P}_{k'}^*}{H_{k0}}\right\}, & P_k^{\text{tmp}} \le 0, \end{cases} \tag{15}$$

$$P_{0,k}^{\max} = \begin{cases} \min\left\{P_T, \dfrac{(1/\lambda_k - P_T) H_{kk} - N_0 - \sum_{k'\neq k,k'=1}^{K} H_{kk'}\widetilde{P}_{k'}^*}{H_{k0}}\right\}, & P_k^{\text{tmp}} > P_T, \\ \min\left\{P_T, \dfrac{(1/\lambda_k) H_{kk} - N_0 - \sum_{k'\neq k,k'=1}^{K} H_{kk'}\widetilde{P}_{k'}^*}{H_{k0}}\right\}, & 0 < P_k^{\text{tmp}} \le P_T, \\ P_T, & P_k^{\text{tmp}} \le 0. \end{cases} \tag{16}$$

Define $\widetilde{\boldsymbol{p}}^* = [\widetilde{P}_1^*, \widetilde{P}_2^*, \dots, \widetilde{P}_k^*, \dots, \widetilde{P}_K^*]^T$. Substituting (10) into (3) and after some mathematical manipulations, we obtain

$$\begin{aligned} U_0(P_0, \widetilde{\boldsymbol{p}}^*) &= \ln\left(1 + \frac{H_{00}P_0}{N_0 + \sum_{k=1}^{K} H_{0k}\widetilde{P}_k^*}\right) - \lambda_0 P_0, \\ &= \ln\left(1 + \frac{H_{00}P_0}{N_0 + \sum_{k=1}^{K} H_{0k}\left\{P_T - \left[P_T - \left(P_k^{\text{tmp}}\right)^+\right]^+\right\}}\right) - \lambda_0 P_0, \\ &= \ln\left(1 + \frac{H_{00}P_0}{N_0 + \sum_{k=1}^{K} H_{0k}\left[P_T - \varepsilon_k' \cdot \left(P_T - \varepsilon_k \cdot P_k^{\text{tmp}}\right)\right]}\right) - \lambda_0 P_0, \end{aligned} \tag{17}$$

where ε_k denotes the indicator function with $\varepsilon_k = 1$ if $P_k^{\text{tmp}} > 0$ and $\varepsilon_k = 0$ otherwise, and ε_k' is the indicator function with $\varepsilon_k' = 1$ if $P_T - (P_k^{\text{tmp}})^+ > 0$ and $\varepsilon_k' = 0$ otherwise. After some further manipulations, the optimization problem for the leader in (4) can be reformulated as follows:

$$\begin{aligned} \max_{P_0}\ & U_0(P_0, \widetilde{\boldsymbol{p}}^*) \\ &= \max_{P_0}\left\{\ln\left(1 + \frac{H_{00}P_0}{A - BP_0}\right) - \lambda_0 P_0\right\}, \\ \text{s.t.}\ & P_0^{\min} \le P_0 \le P_0^{\max}, \end{aligned} \tag{18}$$

where

$$A = N_0 + \sum_{k=1}^{K} H_{0k}\left[P_T - \varepsilon_k' P_T + \varepsilon_k'\varepsilon_k\left(\frac{1}{\lambda_k} - \frac{N_0 + \sum_{k'\neq k,k'=1}^{K} H_{kk'}\widetilde{P}_{k'}^*}{H_{kk}}\right)\right], \tag{19}$$

$$B = \sum_{k=1}^{K} \varepsilon_k'\varepsilon_k \frac{H_{0k}H_{k0}}{H_{kk}}. \tag{20}$$

Then, we have the following theorem.

Theorem 3. *If $\sum_{k=1}^{K} \varepsilon_k\varepsilon_k' \neq 0$, then the optimization problem in (18) has an optimal solution as shown in (21), where*

$$\widetilde{P}_0^* = \begin{cases} \arg\max\left\{U_0(P_0^{\max}, \widetilde{\boldsymbol{p}}^*), U_0(P_0^{\min}, \widetilde{\boldsymbol{p}}^*)\right\}, & C_2^{\ 2} - 4C_1C_3 < 0, \\ \arg\max\left\{U_0(P_0^{\max}, \widetilde{\boldsymbol{p}}^*), U_0(P_0^{\min}, \widetilde{\boldsymbol{p}}^*), U_0(P_0^1, \widetilde{\boldsymbol{p}}^*), U_0(P_0^2, \widetilde{\boldsymbol{p}}^*)\right\}, & C_2^{\ 2} - 4C_1C_3 \ge 0, \end{cases} \tag{21}$$

$$C_1 = \lambda_0 B (H_{00} - B), \tag{22}$$

$$C_2 = \lambda_0 A (2B - H_{00}), \tag{23}$$

$$C_3 = AH_{00} - \lambda_0 A^2, \tag{24}$$

$$P_0^1 = \frac{-C_2 + \sqrt{{C_2}^2 - 4C_1C_3}}{2C_1}, \tag{25}$$

$$P_0^2 = \frac{-C_2 - \sqrt{{C_2}^2 - 4C_1C_3}}{2C_1}. \tag{26}$$

Proof. If $\sum_{k=1}^{K} \varepsilon_k \varepsilon_k' \neq 0$, then $B \neq 0$. Take the first-order derivative of $U_0(P_0, \tilde{\boldsymbol{p}}^*)$ with respect to P_0. Then, we have

$$\frac{\partial U_0(P_0, \tilde{\boldsymbol{p}}^*)}{\partial P_0} = \frac{AH_{00}}{(A - BP_0)^2 + H_{00}P_0(A - BP_0)} - \lambda_0. \tag{27}$$

Set the above expression to zero. Then, we have

$$\begin{aligned} &\lambda_0 B(H_{00} - B)P_0^2 + \lambda_0 A(2B - H_{00})P_0 + AH_{00} \\ &\quad - \lambda_0 A^2 = C_1 P_0^2 + C_2 P_0 + C_3 = 0. \end{aligned} \tag{28}$$

If ${C_2}^2 - 4C_1C_3 < 0$, then (28) has no solution. Correspondingly, the objective function of the optimization problem in (18) is definitely a monotonic function, and its solution must be one of the two endpoints. Then, we have

$$\tilde{P}_0^* = \arg\max \left\{ U_0\left(P_0^{\max}, \tilde{\boldsymbol{p}}^*\right), U_0\left(P_0^{\min}, \tilde{\boldsymbol{p}}^*\right) \right\}. \tag{29}$$

If ${C_2}^2 - 4C_1C_3 \geq 0$, then we can obtain the two solutions of (28), i.e., P_0^1, P_0^2. Since the objective function of the optimization problem in (18) is a continuous function, its solution must be among the extreme points and the endpoints. Correspondingly, we have

$$\begin{aligned} \tilde{P}_0^* = \arg\max \{ &U_0\left(P_0^{\max}, \tilde{\boldsymbol{p}}^*\right), U_0\left(P_0^{\min}, \tilde{\boldsymbol{p}}^*\right), \\ &U_0\left(P_0^1, \tilde{\boldsymbol{p}}^*\right), U_0\left(P_0^2, \tilde{\boldsymbol{p}}^*\right) \}. \end{aligned} \tag{30}$$

This completes the proof. □

3.4. The Proposed Power Control Scheme via Stackelberg Game. We are now ready to develop the proposed power control scheme based on the Stackelberg game described in Algorithm 1. (Note that our proposed scheme can always achieve the optimal solution for any initial point. On the one hand, we analyze theoretically in Section 3.4 that the convergence of the proposed scheme can always be guaranteed. On the other hand, we will prove in Section 3.5 that one and only one SE point exists for the proposed Stackelberg game.) In the proposed scheme, the MUE acts as the leader, the SUEs act as the followers, and the Stackelberg game is formed through the two-layer iteration. In the inner iteration, the SUEs compete with each other, and their own transmit powers are updated iteratively based on the transmit power of the MUE, as shown in Theorem 2. In the outer iteration, the MUE updates its own transmit power based on the transmit powers of the SUEs, as shown in Theorem 3. In the proposed power control scheme, each user plays the best response strategy and maximizes its own utility function in each iteration given the chosen transmit powers of the other users in the previous iteration.

Let $\boldsymbol{W}$ denote a $K \times K$ matrix whose elements are given by

$$\boldsymbol{W}_{kk'} = \begin{cases} \dfrac{H_{kk'}}{H_{kk}}, & k \neq k',\ 1 \leq k,\ k' \leq K, \\ 0, & k = k',\ 1 \leq k,\ k' \leq K. \end{cases} \tag{31}$$

Then, we can establish the following theorem for the convergence of the inner iteration of the proposed scheme.

Theorem 4. *If the matrix norm of $\boldsymbol{W}$ is not larger than 1, i.e., $\|\boldsymbol{W}\| \leq 1$, then the inner iteration of the proposed power control scheme via a Stackelberg game as shown in Algorithm 1 converges.*

Proof. Define

$$\phi_k = P_k^{\text{tmp}}, \tag{32}$$

$$\boldsymbol{\phi}(\boldsymbol{p}) = [\phi_1, \phi_2, \ldots, \phi_k, \ldots, \phi_K]^T, \tag{33}$$

$$\boldsymbol{\mu} = \left[\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots, \frac{1}{\lambda_k}, \ldots, \frac{1}{\lambda_K}\right]^T, \tag{34}$$

$$\begin{aligned} \boldsymbol{\nu} = \Bigg[&\frac{N_0 + H_{10}P_0}{H_{11}}, \frac{N_0 + H_{20}P_0}{H_{22}}, \ldots, \\ &\frac{N_0 + H_{k0}P_0}{H_{kk}}, \ldots, \frac{N_0 + H_{K0}P_0}{H_{KK}} \Bigg]^T. \end{aligned} \tag{35}$$

Then, $\boldsymbol{\phi}(\boldsymbol{p})$ can be expressed in a vector-matrix form as follows:

$$\boldsymbol{\phi}(\boldsymbol{p}) = \boldsymbol{\mu} - \boldsymbol{\nu} - \boldsymbol{W}\boldsymbol{p}. \tag{36}$$

Assume that $\|\boldsymbol{W}\| \leq 1$. Then, we can obtain the following relationship.

$$\left\|\boldsymbol{\phi}(\boldsymbol{p}) - \boldsymbol{\phi}(\boldsymbol{p}')\right\| = \left\|\boldsymbol{W}(\boldsymbol{p} - \boldsymbol{p}')\right\| \tag{37}$$

$$\leq \|\boldsymbol{W}\| \cdot \left\|\boldsymbol{p} - \boldsymbol{p}'\right\| \tag{38}$$

$$\leq \left\|\boldsymbol{p} - \boldsymbol{p}'\right\| \tag{39}$$

According to [18], we know that $\boldsymbol{\phi}(\boldsymbol{p})$ is a contraction. Then, according to the Banach contraction theorem introduced

(i) Step 1: Initialization: $m = 1, n = 1, P_0(1), \widehat{P}_k(1)$ for $1 \le k \le K$.
(ii) Step 2: Update $\widehat{P}_k(m)$ as follows:
$$\widehat{P}_k(m+1) = \frac{1}{\lambda_k} - \frac{N_0 + H_{k0}P_0(n) + \sum_{k' \neq k, k'=1}^{K} H_{kk'}\widehat{P}_{k'}(m)}{H_{kk}},$$
and set $m = m + 1$.
(iii) Step 3: Repeat Step 2 until the inner iteration converges.
(iv) Step 4: According to (10), calculate the transmit power of each SUE, $\widetilde{P}_k^*$, as follows:
$\widetilde{P}_k^* = P_T - \{P_T - [\widehat{P}_k(m)]^+\}^+$.
(v) Step 5: According to (21), calculate the transmit power of the MUE, $\widetilde{P}_0^*(n+1)$, and set $n = n + 1$.
(vi) Step 6: Repeat Steps 2 ~ 5 until the outer iteration converges.

Algorithm 1: The proposed power control scheme via Stackelberg game.

in [18], $\boldsymbol{\phi}(\boldsymbol{p})$ has a unique fixed point that is globally asymptotically stable. Correspondingly, the inner iteration of the proposed power control scheme via the Stackelberg game as shown in Algorithm 1 converges. This completes the proof. □

Note that we can always find a matrix norm of $\boldsymbol{W}$ to satisfy $\|\boldsymbol{W}\| \le 1$. Therefore, the convergence of the inner iteration of the proposed power control scheme can always be guaranteed.

In the following, we briefly analyze the convergence of the outer iteration of the proposed scheme. We know that the utility function U_k is concave with respect to p_k. Therefore, the SUEs can gradually increase their transmit powers from an arbitrary small number to their optima. Then, the obtained transmit power of the SUEs can be used to determine the transmit power of the MUE. When the transmit powers of the SUEs have been increased to their optima, the optimal transmit power of the MUE can then be determined accordingly [19]. For the practical implementation of the proposed Stackelberg game, the SUEs can find their optimal transmit powers by gradually increasing the transmit power until the utility function U_k reaches its maximum due to its concave property. Correspondingly, the MUE can always achieve its SE, i.e., the convergence of the outer iteration of the proposed scheme can always be guaranteed.

3.5. The Existence and Uniqueness of the SE. SE offers a predictable and stable outcome about the transmit power strategies that the MUE and each SUE will choose. For the proposed Stackelberg game, we have the following theorem.

Theorem 5. *One and only one SE point exists for the proposed Stackelberg game.*

Proof. Generally, we can obtain the SE for the proposed Stackelberg game by finding the NE of its subgame. For the proposed Stackelberg game, there is only one leader. Therefore, the best response of the leader can readily be obtained by solving the optimization problem in (4). At the followers' side, the best response can be achieved by solving the optimization problem in (6). Correspondingly, to prove this theorem, we only need to prove that a unique NE point exists for the subgame at the followers' side.

It can be verified that the SUEs' strategy space is a nonempty and closed-bounded convex set in the Euclidean space. Moreover, it can also be verified that the utility function $U_k(P_k, \boldsymbol{p}_{-k}, P_0)$ is continuous with respect to P_k. In addition, the utility function $U_k(P_k, \boldsymbol{p}_{-k}, P_0)$ is concave. According to [20, 21], we know that the NE exists if the players' strategy space is a nonempty and closed-bounded set in the Euclidean space and the utility function is continuous and concave in its strategy space. Correspondingly, the existence of the NE of the subgame at the followers' side can be proven.

Regarding the uniqueness of the NE, we first state the following lemma [22].

Lemma 6. *For a game, if its feasible region is convex and each players' utility function is strictly convex, then the NE of the game is unique.*

Then, according to the above-mentioned discussions and the proof of Theorem 2, we can easily verify the uniqueness of the subgame at the followers' side.

This completes the proof. □

4. Simulation Results

In this section, the performance of the proposed power control scheme via a Stackelberg game is evaluated via simulations. In the simulations, the radii of the macrocell and small cells are set to be 1000 m and 100 m, respectively. The noise spectral density is set to −174 dBm/Hz. Unless otherwise stated, we set $\lambda = \lambda_0 = \lambda_k$, $\forall k$. In the following, for description convenience, we use $\overline{U}_K$, $\overline{R}_K$, and $\overline{P}_K$ to denote the average utility, the average transmit rate, and the average transmit power of the considered SUEs, respectively, and we use $\overline{R}$ to denote the average transmit rate of the considered MUE and SUEs.

In Figures 2 and 3, we show the utility of the MUE and the average utility of the SUEs of the proposed scheme versus the number of iterations with different K for $\lambda = 10^3$ and $P_T = 0$ dBm. As shown, the proposed scheme can converge to a stable state quickly, which verifies that the proposed scheme can converge to the SE. Moreover, both the utility of the MUE and the average utility of the SUEs are observed to

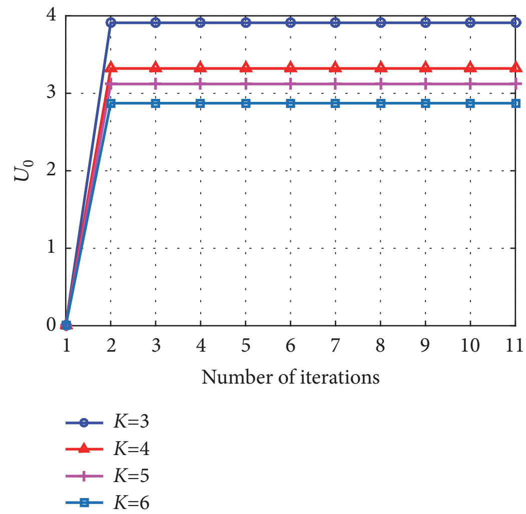

FIGURE 2: The utility of the MUE versus the number of iterations with different K.

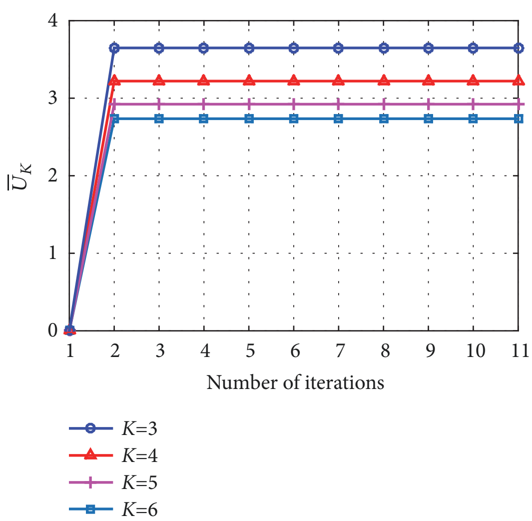

FIGURE 3: The average utility of the SUEs versus the number of iterations with different K.

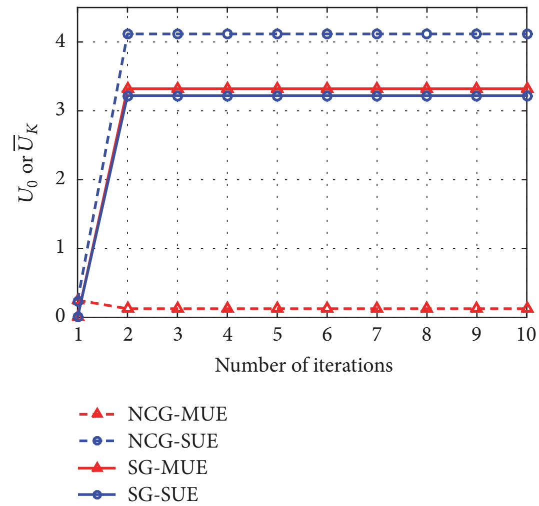

FIGURE 4: The utility versus the number of iterations for different schemes.

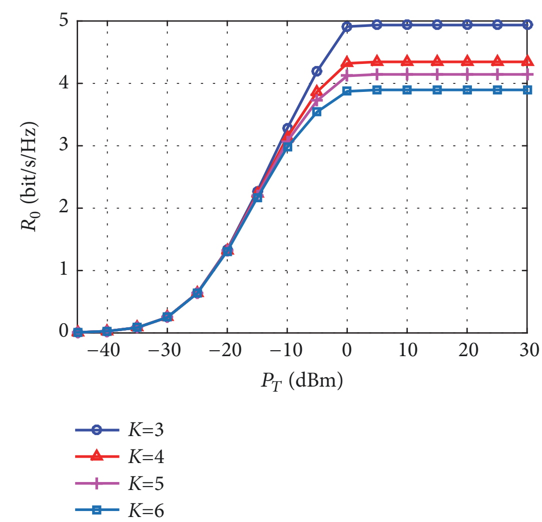

FIGURE 5: The transmit rate of the MUE versus P_T with different K.

decrease with the increased number of the SUEs, which can be attributed to the increased cross-tier or cotier interference.

In Figure 4, we show the performance comparison between our proposed scheme and the noncooperative power control scheme in [23] with $\lambda = 10^3$, $P_T = 0$ dBm, and $K = 4$. For description convenience, the utility of the MUE and the average utility of the SUEs of the proposed Stackelberg-game-based power control scheme are referred to as SG-MUE and SG-SUE, respectively. The utility of the MUE and the average utility of the SUEs of the noncooperative game-based power control scheme in [23] are referred to as NCG-MUE and NCG-SUE, respectively. As shown, the SG-MUE is close to and slightly larger than the SG-SUE, and the NCG-MUE is approximately zero and clearly smaller than the NCG-SUE. This result verifies that the proposed scheme can significantly improve the performance of the MUE. (Note here that the performance improvement is not absolutely free. There is some system overhead between the leader and the followers in order to realize the Stackelberg game in our proposed scheme. However, the quick convergent property of our proposed scheme as illustrated in Figures 2 and 3 indicates that the corresponding system overhead will be affordable.)

In Figures 5 and 6, we show the transmit rate of the MUE and the average transmit rate of the SUEs of the proposed scheme versus P_T with different K for $\lambda = 10^3$. As shown, the transmit rate first increases with P_T when P_T is smaller than a certain threshold value, and then it approaches a steady value. When P_T is sufficiently small, the MUE and the SUEs are constrained by the maximum transmit power. Correspondingly, their transmit rates are relatively small. As P_T increases, their transmit rates increase due to the larger transmit power constraint. When P_T is larger than a certain

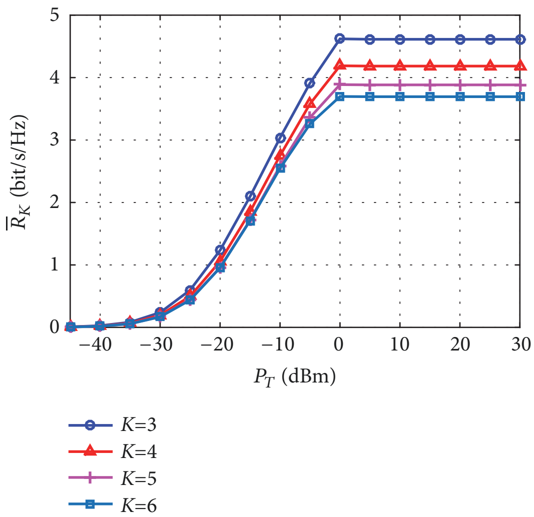

FIGURE 6: The average transmit rate of the SUEs versus P_T with different K.

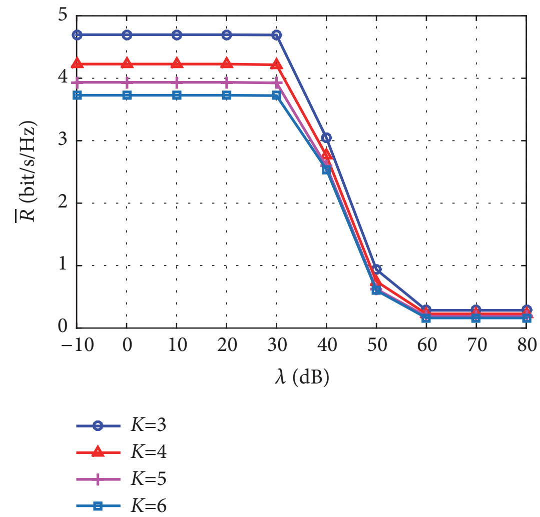

FIGURE 8: The average transmit rate versus λ with different K.

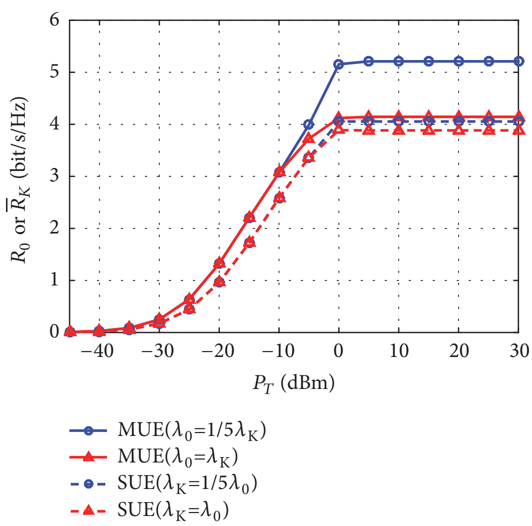

FIGURE 7: The transmit rate versus P_T with different λ_0 and λ_K.

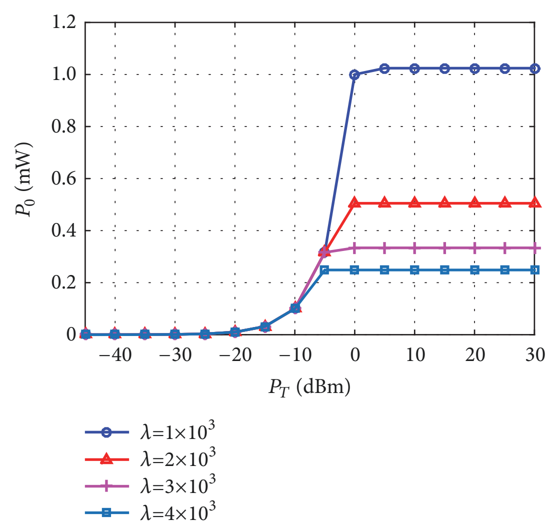

FIGURE 9: The transmit power of the MUE versus P_T with different λ.

threshold value, the transmit power will increase, but it will also simultaneously cause more interference. Therefore, the transmit rate of the MUE and that of the SUEs will both stop increasing.

In Figure 7, we show the transmit rate of the MUE and the average transmit rate of the SUEs versus P_T with different λ_0 and λ_K for $K = 4$. As shown, the transmit rate of the MUE (the average transmit rate of the SUEs) is larger when λ_0 (λ_K) is relatively small. The reason for this result is that a smaller cost of the transmit power will stimulate the corresponding player to employ a relatively large transmit power, subsequently resulting in a larger transmit rate.

In Figure 8, we show the average transmit rate of the MUE and SUEs of the proposed scheme versus λ for $K = 4$ and $P_T = 0$ dBm. As shown, the average transmit rate remains at a high value when λ is smaller than 30 dB, decreases gradually with the increased λ, and finally remains at a small value. The reason for this behavior is that the MUE or SUE will choose to decrease the transmit power and also the corresponding transmit rate with the increased λ.

In Figures 9 and 10, we show the transmit power of the MUE and the average transmit power of the SUEs versus P_T with different λ for $K = 4$. As shown, the transmit power decreases with λ. Moreover, the transmit power increases with P_T when P_T is smaller than 0 dBm and remains approximately constant when P_T is larger than 0 dBm. The reason for this result is that the maximum transmit power constraint will have no influence on the power control with a sufficiently large P_T.

5. Conclusions

In this paper, we have formulated a power control Stackelberg game for two-tier small-cell networks by considering both the transmit rate and cost. The optimal transmit powers

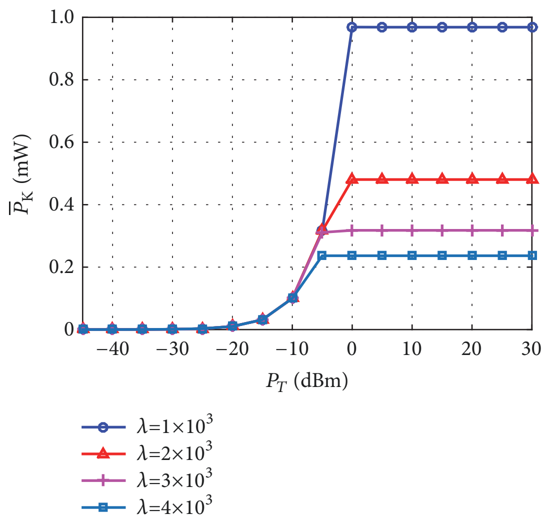

FIGURE 10: The average transmit power of the SUEs versus P_T with different λ.

of the MUE and SUEs have been obtained based on the backward induction method. We have developed a two-layer iterative power control scheme and proven the convergence of this scheme. We have also shown the existence and uniqueness of the SE in the formulated Stackelberg game. Numerical results have been presented to demonstrate the desirable performance of the proposed scheme. For future work, we would like to explore power control with incomplete information for two-tier small-cell networks.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] J. Hoydis, M. Kobayashi, and M. Debbah, "Green small-cell networks," *IEEE Vehicular Technology Magazine*, vol. 6, no. 1, pp. 37–43, 2011.

[2] N. Lu, Y. Jiang, F. Zheng, and X. You, "Energy efficient power control for the two-tier networks with small cells and massive MIMO," in *Proceedings of the IEEE Wireless Communications and Networking Conference Workshops*, pp. 127–132, April 2016.

[3] Y. Jiang, Q. Liu, F. Zheng, X. Gao, and X. You, "Energy-efficient joint resource allocation and power control for D2D communications," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 8, pp. 6119–6127, 2016.

[4] J. Zheng, Y. Cai, Y. Liu, Y. Xu, B. Duan, and X. Shen, "Optimal power allocation and user scheduling in multicell networks: base station cooperation using a game-theoretic approach," *IEEE Transactions on Wireless Communications*, vol. 13, no. 12, pp. 6928–6942, 2014.

[5] N. Sawyer and D. B. Smith, "Flexible resource allocation in device-to-device communications using stackelberg game theory," *IEEE Transactions on Wireless Communications*, vol. 67, no. 1, pp. 653–667, 2019.

[6] J. Zhang, Y. Wu, N. Zhang et al., "Optimal power control in ultra-dense small cell networks: a game-theoretic approach," *IEEE Transactions on Wireless Communications*, vol. 16, no. 7, pp. 4139–4150, 2017.

[7] Y. Jiang, N. Lu, Y. Chen et al., "Energy efficient non-cooperative power control in small cell networks," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 8, pp. 7540–7547, 2017.

[8] C. Yang, H. Dai, J. Li, Y. Zhang, and Z. Han, "Distributed interference-aware power control in ultra-dense small cell networks: a robust mean field game," *IEEE Access*, vol. 6, pp. 12 608–12 619, 2018.

[9] Y. A. Al-Gumaei, K. A. Noordin, A. W. Reza, and K. Dimyati, "A new power control game in two-tier femtocell networks," in *Proceedings of the IEEE International Conference on Telematics and Future Generation Networks*, May 2015.

[10] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for spectrum-sharing femtocell networks: a stackelberg game approach," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 538–549, 2012.

[11] P. Li and Y. Zhu, "Price-based power control of femtocell networks: a Stackelberg game approach," in *Proceedings of the 2012 IEEE 23rd International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC 2012*, pp. 1185–1191, September 2012.

[12] D. Fudenberg and J. Tirole, *Game Theory*, The MIT Press, 1993.

[13] M. Bennis, M. Debbah, S. Lasaulce, and A. Anpalagan, "A hierarchical game approach to inter-operator spectrum sharing," in *Proceedings of the IEEE Global Telecommunications Conference*, December 2009.

[14] M. Bennis, M. Le Treust, S. Lasaulce, M. Debbah, and J. Lilleberg, "Spectrum sharing games on the interference channel," in *Proceedings of the International Conference on Game Theory for Networks*, pp. 515–522, May 2009.

[15] Y. Li, L. Xiao, J. Liu, and Y. Tang, "Power control Stackelberg game in cooperative anti-jamming communications," in *Proceedings of the 5th International Conference on Game Theory for Networks*, february 2014.

[16] R. Gibbons, *A Primer in Game Theory*, Pearson Academic, 1992.

[17] Z. Su, Q. Xu, F. Hou, Q. Yang, and Q. Qi, "Edge caching for layered video contents in mobile social networks," *IEEE Transactions on Multimedia*, vol. 19, no. 10, pp. 2210–2221, 2017.

[18] K. Shum, K. Leung, and C. Sung, "Convergence of iterative waterfilling algorithm for gaussian interference channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 6, pp. 1091–1100, 2007.

[19] B. Wang, Z. Han, and K. J. R. Liu, "Distributed relay selection and power control for multiuser cooperative communication networks using stackelberg game," *IEEE Transactions on Mobile Computing*, vol. 8, no. 7, pp. 975–990, 2009.

[20] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[21] T. Alpcan and T. Basar, "A game-theoretic framework for congestion control in general topology networks," in *Proceedings of the IEEE 41st Conference on Decision and Control*, December 2002.

[22] B. Wang, Y. Wu, and K. J. R. Liu, "Game theory for cognitive radio networks: an overview," *Computer Networks*, vol. 54, no. 14, pp. 2537–2561, 2010.

[23] V. N. Ha and L. B. Le, "Distributed base station association and power control for heterogeneous cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 1, pp. 282–296, 2014.