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Dual Methods for Optimal Allocation of Telecommunication Network Resources with Several Classes of Users

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Abstract: We consider a general problem of optimal allocation of limited resources in a wireless telecommunication network. The network users are divided into several different groups (or classes), which correspond to different levels of service. The network manager must satisfy these different users' requirements. This approach leads to a convex optimization problem with balance and capacity constraints. We present several decomposition type methods to find a solution to this problem, which exploit its special features. We suggest applying first the dual Lagrangian method with respect to the total capacity constraint, which gives the one-dimensional dual problem. However, calculation of the value of the dual cost function requires solving several optimization problems. Our methods differ in approaches for solving these auxiliary problems. We consider three basic methods: Dual Multi Layer (DML), Conditional Gradient Dual Multilayer (CGDM) and Bisection (BS). Besides these methods we consider their modifications adjusted to different kind of cost functions. Our comparison of the performance of the suggested methods on several series of test problems show satisfactory convergence. Nevertheless, proper decomposition techniques enhance the convergence essentially.

Keywords: telecommunication networks; wireless networks; service levels; resource allocation; optimization problem; decomposition methods; Lagrange duality

1. Introduction

Efficient allocation of limited resources in communication networks require flexible mechanisms, which are based on proper mathematical models, since the conventional fixed allocation rules may lead to congestion effects and additional expenses from the inefficient utilization of network resources; see e.g., [1–3]. In particular, spectrum sharing is now one of the most critical issues and various adaptive mechanisms for allocation of resources in wireless telecommunication networks have been suggested. Most papers in this field are devoted to game-theoretic models and implementation of decentralized iterative methods for finding the Nash equilibrium points or their generalizations; see e.g., [4,5]. At the same time, various optimization-based mechanisms are also suggested; see e.g., [3,5–7]. Further, the allocation of energy and computing resources are considered in [8,9]. Management of these highly complicated systems are often based on proper decomposition approaches, which can involve zonal, time, frequency and other decomposition techniques.

In [10–15], several optimal resource allocation problems in telecommunication networks and proper zonal decomposition-based methods were suggested. They assumed that the network manager can satisfy all the varying user requirements or can buy additional volumes of the resource. This approach leads to constrained convex optimization problem for some selected time period. However, these models do not take into account possible differentiation of users with respect to service levels, which yields different service costs and somewhat different optimization problems.

In this paper, we consider problems of optimal allocation of a homogeneous resource in a telecommunication network with the differentiation of users. In such a way, we give a new formulation of this problem as an optimization problem and present several dual decomposition type methods for the affine and convex cases. We also compare the performance of the suggested methods on several series of test problems. The comparison shows the advantage of proposed methods over the conventional ones.

2. Problem Formulation

Let us consider a single telecommunication network with nodes (users). A network manager offers users m levels of network service (classes), which is reflected by expenses and prices. Within some selected time period, the network manager can offer a limited total amount C of a homogeneous resource of the network. An amount of resource allocated to the i-th class service is supposed to be equal to $\varphi_i(x_i)$ if x_i is an unknown consumed traffic volume at this level ($0 \leq x_i \leq \beta_i$). The cost of implementation (network expense) of the amount x_i of the i-th service level is supposed to be equal to $\mu_i(x_i)$. Each user can choose only one level of service. Let $N = \{1, \dots, n\}$ denote a set of users, and N_i a set of users of the i-th class (level) for $i = 1, \dots, m$. Let y_j denote the unknown traffic volume offered to the j-th user with $0 \leq y_j \leq \alpha_j$ and $\eta_j(y_j)$ is the fee (incentive) value paid by the j-th user for this traffic. If all the users are attributed to the classes, we can calculate the total traffic volume for each i-th level as follows:

$$x_i = \sum_{j \in N_i} y_j.$$

The general problem of the network manager is to find an optimal allocation of the limited homogeneous resource among the users in order to maximize the total payment received from the users and minimize the total network implementation expenses. This problem is now formulated as follows:

$$\max_{(x,y) \in W, \sum_{i=1}^{m} \varphi_i(x_i) \leq C} \to f(x,y), \tag{1}$$

where

$$f(x,y) = \sum_{i=1}^{m} \left[\sum_{j \in N_i} \eta_j(y_j) - \mu_i(x_i) \right] \tag{2}$$

and

$$W = \left\{ (x,y) \,\middle|\, x_i = \sum_{j \in N_i} y_j,\ 0 \leq y_j \leq \alpha_j,\ j \in N_i,\ 0 \leq x_i \leq \beta_i, i = 1, \dots, m \right\}. \tag{3}$$

In what follows we shall suppose that all the functions $\mu_i(x_i)$, $\varphi_i(x_i)$ and $-\eta_j(y_j)$ are convex, then (1)–(3) is a convex optimization problem.

3. Solution Methods

It is well known that many efficient solution methods for convex optimization problems exist; see e.g., [16,17]. However, due to large dimensionality and inexact data of the optimal resource allocation problems in telecommunication networks one can meet serious difficulties when solving these problems with conventional general iterative solution methods. To create an efficient method just for problem (1)–(3), we have to take into account its separability and apply certain decomposition approach. Moreover, the standard duality scheme using the Lagrangian function with respect to all the functional constraints leads to the multi-dimensional dual optimization problem. We will apply another approach, which was suggested in [12,18] and leads to solution of one-dimensional problems.

Let us first define the Lagrange function of problem (1)–(3) as follows:

$$L(x,y,\lambda) = f(x,y) - \lambda \left(\sum_{i=1}^{m} \varphi_i(x_i) - C \right).$$

This means we will utilize the Lagrangian multiplier λ only for the total resource bound. We can now replace problem (1)–(3) with its dual:

$$\min_{\lambda \geq 0} \to \psi(\lambda), \tag{4}$$

where

$$\psi(\lambda) = \max_{(x,y)\in W} L(x,y,\lambda) = \lambda C + \max_{(x,y)\in W} \sum_{i=1}^{m} \left[\sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda\varphi_i(x_i)) \right].$$

By duality (see e.g., [16,17]), problems (1)–(3) and (4) have the same optimal value. However, solution of (4) can be found by one of the well-known single-dimensional optimization algorithms; see e.g., [17]. The main problem is to implement these algorithms properly.

To calculate the value of $\psi(\lambda)$ we have to solve the inner problem:

$$\max_{(x,y)\in W} \to \sum_{i=1}^{m} \left[\sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda\varphi_i(x_i)) \right].$$

This problem clearly decomposes into m independent class problems

$$\max \to \sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda\varphi_i(x_i)), \tag{5}$$

subject to

$$x_i = \sum_{j\in N_i} y_j,\ 0 \leq y_j \leq \alpha_j,\ j \in N_i,\ 0 \leq x_i \leq \beta_i,\ \text{ for } i = 1,\ldots,m. \tag{6}$$

Our methods for problem (1)–(3) will differ in approaches to problem (5)–(6).

We first describe the decomposition approach, which follows in general that from [10–12]. Denote by $\nu_i(x_i)$ the optimal value of the i-th service optimization problem:

$$\max \to \sum_{j\in N_i} \eta_j(y_j) \tag{7}$$

subject to

$$\sum_{j\in N_i} y_j = x_i, 0 \leq y_j \leq \alpha_j, j \in N_i. \tag{8}$$

Then (5)–(6) reduces to the one-dimensional problem:

$$\min_{0\leq x_i\leq\beta_i} \to \nu_i(x_i) - \mu_i(x_i) - \lambda\varphi_i(x_i). \tag{9}$$

It is easy to see that $\nu_i(x_i)$ is a convex, but non differentiable function in general.

Thus, the initial problem (1)–(3) is replaced by its one-dimensional dual (4) with the cost function $\psi(\lambda)$, such that calculation of its value reduces to solution of m independent problems of form (5)–(6), whose calculation again reduces to solution of one-dimensional problems of form (9).

However, each function ν_i is given algorithmically, i.e., via solution of problem (7)–(8). In the general case we can apply again a dual type method to find the value of $\nu_i(x_i)$. Let us to introduce the the Lagrange function

$$\tilde{L}_j(y, \theta_i) = \sum_{j \in N_i} \eta_j(y_j) - \theta_i \left[\sum_{j \in N_i} y_j - x_i \right],$$

and then to solve the one-dimensional dual:

$$\min_{\theta_i \geq 0} \to \zeta_i(\theta_i),$$

where

$$\zeta_i(\theta_i) = \theta_i x_i + \sum_{j \in N_i} \max_{0 \leq y_j \leq \alpha_j} [\eta_j(y_j) - \theta_i y_j].$$

Therefore, we can use here only algorithms for a set of hierarchical one-dimensional problems. Let us denote this method as (**DML**).

Please note that this approach involves several levels of hierarchical problems requiring certain concordance in the accuracies of the solution of all these problems, besides, each solution of one upper level problem requires solution of all the lower level problems many times, which entails large computational costs. However, they can be reduced for some special types of functions.

For instance, consider the case where the functions $\eta_j(y_j)$, $j \in N_i$ are affine, whereas the functions $\varphi_i(x_i)$ and $\mu_i(x_i)$ are convex and differentiable. Then we can find an exact solution of problems (7) and (8) by a simple ordering algorithm in a finite number of iterations; see [19] for more detail.

Next, consider the particular case where all the functions $\eta_j(y_j)$, $\mu_i(x_i)$, and $\varphi_i(x_i)$ are affine, i.e.,

$$\begin{aligned}
\eta_j(y_j) &= \eta_{j,1} y_j + \eta_{j,0}, \ \eta_{j,1} > 0, j \in N_i, i = 1, \ldots, m, \\
\mu_i(x_i) &= \mu_{i,1} x_i + \mu_{i,0}, \ \mu_{i,1} > 0, i = 1, \ldots, m, \\
\varphi_i(x_i) &= \varphi_{i,1} x_i + \varphi_{i,0}, \ \varphi_{i,1} > 0, i = 1, \ldots, m.
\end{aligned} \tag{10}$$

Then the cost function in (5) can be rewritten equivalently as

$$\eta_{j,1} y_j - (\mu_{i,1} + \lambda \varphi_{i,1}) x_i.$$

This means that problems (5) and (6) reduces to a two-side auction market with fixed prices (see [19]) and also is solved in a finite number of iterations by a simple ordering algorithm; see also [13,18]. Let us denote this method as (**SDM**).

We can extend this approach to the case where the functions $\eta_j(y_j)$ are affine as in (10), whereas the functions $\varphi_i(x_i)$ and $\mu_i(x_i)$ are only convex and differentiable. This means that the prices (marginal utilities) $\eta_{j,1}$ of the users are fixed, but the marginal expenses and prices depend on volumes, so that they are non-decreasing.

Set $y(i) = (y_j)_{j \in N_i}$ and

$$W_i = \left\{ (x_i, y(i)) \middle| x_i = \sum_{j \in N_i} y_j, \ 0 \leq y_j \leq \alpha_j, \ j \in N_i, \ 0 \leq x_i \leq \beta_i \right\}.$$

The necessary and sufficient optimality condition for problem (5) and (6) is now written in the form of the variational inequality: find $(\bar{x}_i, \bar{y}(i)) \in W_i$ such that

$$(\mu_i'(\bar{x}_i) + \lambda \varphi_i'(\bar{x}_i))(x_i - \bar{x}_i) - \sum_{j \in N_i} \eta_{j,1}(y_j - \bar{y}_j) \geq 0, \quad \forall (x_i, y(i)) \in W_i. \tag{11}$$

This is a two-sided market equilibrium problem with one seller and several buyers; see e.g., [19]. It is equivalent to the problem of finding a vector $(\bar{x}_i, \bar{y}(i)) \in W_i$ and a cutting price $\bar{p}_i$ such that

$$\mu_i'(\bar{x}_i) + \lambda\varphi_i'(\bar{x}_i) \begin{cases} \geq \bar{p}_i, & \text{if } \bar{x}_i = 0, \\ = \bar{p}_i, & \text{if } \bar{x}_i \in [0, \beta_i], \\ \leq \bar{p}_i, & \text{if } \bar{x}_i = \beta_i, \end{cases} \tag{12}$$

and

$$\eta_{j,1} \begin{cases} \leq \bar{p}_i, & \text{if } \bar{y}_j = 0, \\ = \bar{p}_i, & \text{if } \bar{y}_j \in [0, \alpha_j], \\ \leq \bar{p}_i, & \text{if } \bar{y}_j = \alpha_j. \end{cases} \tag{13}$$

Since buyers prices are fixed, we can re-arrange them to be non-increasing and then find easily an intersection point of the staircase-wise inverse common demand and offer price $\mu_i'(x_i) + \lambda\varphi_i'(x_i)$ lines; see also [13]. Therefore, the exact solution of problem (11) or (5) and (6) can be also found directly by simple ordering type algorithms applying to (12) and (13) although (5) and (6) contains a non-linear function. In other words, calculation of values of $\psi(\lambda)$ can be now accomplished by several independent simple ordering type algorithms. Notice that the re-arrangement of bid prices $\eta_{j,1}$ in each class should be made only one time that reduces the computational expenses essentially in comparison with the general duality approach. Let us denote this method also as (**SDM**).

We now again consider the general case where all the functions $\mu_i(x_i)$, $\varphi_i(x_i)$, and $-\eta_j(y_j)$ are convex and differentiable. For these problems there exist many rather efficient solution methods; see e.g., [20] and references therein. In view of the above properties we can replace each problem (5) and (6) with a sequence of linearized problems of the form:

$$\min_{(x_i, y(i)) \in W_i} \to \left[(\mu_i'(x_i^k) + \lambda\varphi_i'(x_i^k))x_i - \sum_{j \in N_i} \eta_j'(y_j^k) y_j \right] \tag{14}$$

if we apply the conventional conditional gradient method (**CGM**) as suggested in [21]. For the sake of clarity, we describe (**CGM**) applied to the general optimization problem

$$\min_{v \in V} \to \phi(v),$$

where V is a convex closed set, ϕ is a convex and differentiable function.

(**CGM**) Take an arbitrary initial point $v^0 \in V$ and a number $\delta > 0$. At the s-th iteration, $s = 0, 1, \ldots$, we have a point $v^s \in V$ and calculate $u^s \in V$ as a solution of the linear programming problem

$$\min_{u \in V} \to \langle \phi'(v^s), u \rangle. \tag{15}$$

Then we set $p^s = u^s - v^s$. If $\|p^s\| \geq -\delta$, stop, we have an approximate solution. Otherwise we find the next iterate v^{s+1} as follows:

$$v^{s+1} = \sigma_s u^s + (1 - \sigma_s) v^s,$$

where $\sigma_s \in (0, 1)$ is a step-size parameter.

In particular, we can utilize the inexact line search procedure: Find m as the minimal non-negative integer such that

$$\phi(v^s + \theta^m p^s) \leq \phi(v^s) + \alpha\theta^m \langle \phi'(v^s), p^s \rangle,$$

for some $\alpha \in (0, 1)$ and $\theta \in (0, 1)$, and set $\sigma_s = \theta^m$; see [22].

It is easy to see that (15) gives then (14). Hence, (14) can be solved by simple ordering type algorithms as in (**SDM**). We denote this method as (**CGDM**). However, this approach requires application of (**CGM**) many times at each iteration of a single-dimensional optimization algorithm applied to the upper problem (4). At the same time, we can apply the same dual decomposition method to problem (5) and (6). For the sake of simplicity, we rewrite (5) and (6) as follows:

$$\max_{(x,y)\in D} \to \sum_{j\in J} \eta_j(y_j) - u(x), \tag{16}$$

where

$$D = \left\{ (x,y) \,\middle|\, x = \sum_{j\in J} y_j,\ 0 \le y_j \le \alpha_j,\ j \in J,\ 0 \le x \le \beta \right\},$$
$$x = x_i,\ y = (y_j)_{j\in J},\ J = N_i,\ \beta = \beta_i,$$
$$u(x) = \mu_i(x) + \lambda\varphi_i(x).$$

Let $g(x) = u'(x)$ and $w_j(y_j) = \eta_j'(y_j)$. The necessary and sufficient optimality condition for problem (16) is given in (11) and re-written now as the variational inequality: find $(\bar{x}, \bar{y}) \in D$ such that

$$g(\bar{x})(x - \bar{x}) - \sum_{j\in J} w_j(\bar{y}_j)(y_j - \bar{y}_j) \ge 0, \forall (x,y) \in D.$$

The optimality conditions in (12) and (13) have the form: find $(\bar{x}, \bar{y}) \in D$ and p^* such that

$$g(\bar{x}) \begin{cases} \ge p^*, & \text{if } \bar{x}_i = 0, \\ = p^*, & \text{if } \bar{x}_i \in [0, \beta], \\ \le p^*, & \text{if } \bar{x}_i = \beta, \end{cases} \tag{17}$$

and

$$w_j(\bar{y}_j) \begin{cases} \le p^*, & \text{if } \bar{y}_j = 0, \\ = p^*, & \text{if } \bar{y}_j \in [0, \alpha_j], \quad \text{for } j \in J. \\ \ge p^*, & \text{if } \bar{y}_j = \alpha_j; \end{cases} \tag{18}$$

Following the dual approach, we write the Lagrange function of problem (16) with the negative sign:

$$\begin{aligned} M(x,y,p) &= u(x) - \sum_{j\in J} \eta_j(y_j) - p\left(x - \sum_{j\in J} y_j\right) \\ &= (u(x) - px) - \sum_{j\in J} (\eta_j(y_j) - py_j). \end{aligned}$$

To find a value of the dual cost function

$$\theta(p) = \min_{x\in[0,\beta],\ y\in[0,\alpha]} M(x,y,p),$$

where $\alpha = (\alpha_j)_{j\in J}$, we have to solve the one-dimensional problems:

$$\min_{0\le x\le\beta} \to (u(x) - px),$$

and

$$\min_{0\le y_j\le\alpha_j} \to (-\eta_j(y_j) + py_j), \quad \text{for } j \in J.$$

For the sake of simplicity, we also suppose that the functions u and $-\eta_j$ are strictly convex. Then solutions of the above problems denoted by $x(p)$ and $y_j(p)$, $j \in J$, respectively, are defined uniquely. It follows that the function $\theta(p)$ is concave and differentiable with the derivative

$$\theta'(p) = \sum_{j \in J} y_j(p) - x(p).$$

Besides, the one-dimensional dual problem

$$\max_{p} \to \theta(p)$$

coincides with the simple equation

$$\theta'(p) = 0, \tag{19}$$

where $\theta'(p)$ is non-increasing. Therefore, if p^* is the solution of (19), then we can find the solution of problem (16) from (17) to (18) by setting $p = p^*$, which gives a solution of the initial problem (5) and (6).

To find a solution of (17) we can apply bisection type algorithms. Let $\gamma' = g(0)$ and $\gamma'' = g(\beta)$. Then $\gamma' < \gamma''$. Let $\delta'_j = w(0)$ and $\delta''_j = w(\alpha_j)$. If we set $p'' = \max_{j \in J} \delta'_j$ and $p' = \gamma'$, then the case $p'' \leq p'$ gives immediately the zero solutions in accordance with (17) and (18). So we can consider only the non-trivial case where $p' < p''$. Then by (17) and (18) we must have $\theta'(p') \geq 0$ and $\theta'(p'') \leq 0$. These properties enable us to utilize the simplest bisection algorithm; see e.g., [14,15].

Algorithm (BS). Given an accuracy $\varepsilon > 0$ and the initial segment $[p', p'']$, we take $\tilde{p} = 0.5(p' + p'')$, calculate $\theta'(\tilde{p})$. Then we set $p' = \tilde{p}$ if $\theta'(\tilde{p}) > 0$ and $p' = \tilde{p}$ otherwise, until $(p'' - p') < \varepsilon$.

Also, if all the functions are quadratic, we can utilize a heuristic method similar to that in [14].

Algorithm (SQ). Let $\omega_j = \eta_{j,1} + \lambda\varphi_{i,1}$. Define $J_a = \{j \in J \mid \omega_j > p'\}$, set $y^*_j = 0$ for $j \notin J_a$ and re-arrange the indices in J_a to have the descending order for the values of ω_j. Then find two sequential indices j_l and j_{l+1} in J_a such that $\Delta_l < 0$ and $\Delta_{l+1} > 0$, where

$$\Delta_l = \sum_{s=1}^{l} y_{j_s}(\omega_{j_l}) - x\omega_{j_l}).$$

Then find p^* such that $\theta'(p^*) = 0$ in the segment $[\omega_{j_l}, \omega_{j_{l+1}}]$.

4. Numerical Experiments

The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial interval for the dual variable λ were chosen as $[0, 1000]$. The parameters $\beta_i (i = 1, \ldots, m)$ and $\alpha_j (j \in N_i, i = 1, \ldots, m)$ were chosen as values of trigonometric functions in $[1, 51]$ and $[1, 2]$, respectively. We set the constant C to be equal 1000. The number of classes was varied from 3 to 45, the number of users was varied from 210 to 1010. Users were distributed among classes either uniformly or according to the normal distribution.

We considered the following kinds of functions in test problems of form (1) to (3):

[Case L] All the functions $\mu_i(x_i)$, $\varphi_i(x_i)$, and $\eta_j(y_j)$ are affine;
[Case QL] All the functions $\eta_j(y_j)$ are affine, all the functions $\mu_i(x_i)$ and $\varphi_i(x_i)$ are quadratic;
[Case EQ] All the functions $-\eta_j(y_j)$ are convex quadratic, all the functions $\mu_i(x_i)$ and $\varphi_i(x_i)$ are convex exponential;
[Case Q] All the functions $-\eta_j(y_j)$, $\mu_i(x_i)$, and $\varphi_i(x_i)$ are convex quadratic;
[Case E] All the functions $-\eta_j(y_j)$, $\mu_i(x_i)$, and $\varphi_i(x_i)$ are convex exponential;
[Case LG] All the functions $-\eta_j(y_j)$, $\mu_i(x_i)$, and $\varphi_i(x_i)$ are convex logarithmic.

Let J denote the total number of users. The test functions were determined as follows:

1. Linear functions

$$\eta_j(y_j) = \eta_{j,1}y_j + \eta_{j,0}, \eta_{j,1} > 0,\ j = 1, \ldots, J,$$
$$\mu_i(x_i) = \mu_{i,1}x_i + \mu_{i,0}, \mu_{i,1} > 0,\ i = 1, \ldots, m,$$
$$\varphi_i(x_i) = \varphi_{i,1}x_i + \varphi_{i,0}, \varphi_{i,1} > 0,\ i = 1, \ldots, m,$$

where

$$\eta_{j,1} = 2|\sin(j+1)| + 1, \eta_{j,0} = 2|\sin(2j)| + 1,\ j = 1, \ldots, J,$$
$$\mu_{i,1} = |\cos(i)| + 1, \mu_{i,0} = 2|\cos(2i)| + 1,\ i = 1, \ldots, m,$$
$$\varphi_{i,1} = \mu_{i,1}, \varphi_{i,0} = \mu_{i,0},\ i = 1, \ldots, m.$$

2. Quadratic functions

$$\eta_j(y_j) = 0.5\eta_{j,2}y_j^2 + \eta_{j,1}y_j, \eta_{j,2} < 0,\ j = 1, \ldots, J,$$
$$\mu_i(x_i) = 0.5\mu_{i,2}x_i^2 + \mu_{i,1}x_i, \mu_{i,2} > 0,\ i = 1, \ldots, m,$$
$$\varphi_i(x_i) = 0.5\varphi_{i,2}x_i^2 + \varphi_{i,1}x_i, \varphi_{i,2} > 0,\ i = 1, \ldots, m,$$

where

$$\eta_{j,2} = -4|\cos(2j-1)| - 4, \eta_{j,1} = |\sin(j+1)| + 1,\ j = 1, \ldots, J,$$
$$\mu_{i,2} = |\sin(2i)| + 1, \mu_{i,1} = |\cos(i)| + 3,\ i = 1, \ldots, m,$$
$$\varphi_{i,2} = \mu_{i,2}, \varphi_{i,1} = \mu_{i,1},\ i = 1, \ldots, m.$$

3. Exponential functions

$$\eta_j(y_j) = \eta_{j,0} + \eta_{j,1}y_j - \eta_{j,2}e^{\eta_{j,3}y_j}, \eta_{j,1}, \eta_{j,2} > 0,\ j = 1, \ldots, J,$$
$$\mu_i(x_i) = \mu_{i,0}e^{\mu_{i,1}x_i}, \mu_{i,1}, \mu_{i,0} > 0,\ i = 1, \ldots, m,$$
$$\varphi_i(x_i) = \varphi_{i,0}e^{\varphi_{i,1}x_i}, \varphi_{i,1}, \varphi_{i,0} > 0,\ i = 1, \ldots, m,$$

where

$$\eta_{j,3} = |\sin(j+1)| + 1, \eta_{j,2} = 2|\sin(2j)| + 1,$$
$$\eta_{j,1} = 2|\sin(j+1)| + 8, \eta_{j,0} = 2|\sin(2j)| + 9,\ j = 1, \ldots, J,$$
$$\mu_{i,1} = |\cos(i)| + 1, \mu_{i,0} = 2|\cos(2i)| + 1,\ i = 1, \ldots, m,$$
$$\varphi_{i,1} = \mu_{i,1}, \varphi_{i,0} = \mu_{i,0},\ i = 1, \ldots, m.$$

4. Logarithmic functions

$$\eta_j(y_j) = \eta_{j,2}\ln(1 + \eta_{j,0} + \eta_{j,1}y_j), \eta_{j,0}, \eta_{j,1}, \eta_{j,2} > 0,\ j = 1, \ldots, J,$$
$$\mu_i(x_i) = \mu_{i,0} + \mu_{i,1}x_i - \ln(1 + \mu_{i,2} + \mu_{i,3}x_i), \mu_{i,1}, \mu_{i,2}, \mu_{i,3} > 0,\ i = 1, \ldots, m,$$
$$\varphi_i(x_i) = \varphi_{i,0} + \varphi_{i,1}x_i - \ln(1 + \varphi_{i,2} + \varphi_{i,3}x_i), \varphi_{i,1}, \varphi_{i,2}, \varphi_{i,3} > 0,\ i = 1, \ldots, m,$$

where

$$\eta_{j,0} = 2|\sin(2j)|, \eta_{j,1} = |\sin(j+1)| + 1,$$
$$\eta_{j,2} = 3|\sin(2j)| + 1,\ j = 1, \ldots, J,$$
$$\mu_{i,0} = 2|\cos(2i)| + 1, \mu_{i,1} = |\cos(i)| + 1,$$
$$\mu_{i,2} = 2|\cos(2i)|, \mu_{i,3} = |\cos(i)| + 1,\ i = 1, \ldots, m,$$
$$\varphi_{i,0} = \mu_{i,0}, \varphi_{i,1} = \mu_{i,1}, \varphi_{i,2} = \mu_{i,2}, \varphi_{i,3} = \mu_{i,3},\ i = 1, \ldots, m.$$

For all the methods of solving problem (1)–(3) the accuracy of solution of upper dual problem (4) was varied from 10^{-1} to 10^{-4}. The accuracy of solution of lower level problems was fixed to be equal 10^{-2}. For each set of the parameters 50 tests were made. The aim of the numerical experiments is to calculate the time complexity (total processor time) of the methods with different kind of cost functions. In the tables, T_ε denotes the total processor time in seconds. The averaged results of computations for Case L are given in Tables 1–3, for Case QL are given in Tables 4–6, for Case Q are given in Tables 7–9, for Case EQ are given in Tables 10–12, for Case E are given in Tables 13–15, for Case LG are given in Tables 16–18.

Together with the three basic methods for problem (1)–(3) named (**DML**), (**CGDM**), and (**BS**), we tested also their modifications adjusted mainly to some particular classes of problems. We applied the method (**DML**) with adaptive strategy of choosing the inner accuracies and named it (**DMLA**). In the case where the functions $\eta_j(y_j)$ are affine, we applied also the simplified versions of these methods named (**DMLS**) and (**DMLAS**), respectively. They solve auxiliary problems (7) and (8) by a simple ordering algorithm in a finite number of iterations and require only one arrangement of buyers' prices. Methods (**DML**), (**DMLA**), (**DMLS**), (**DMLAS**) and (**SDM**) were applied for cases L and QL, where (**DMLS**) and (**DMLAS**) showed better performance than (**DML**) and (**DMLA**), but (**SDM**) showed the best results here.

Table 1. Results for Case L with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
10^{-1}	0.1680	0.0897	0.0025	0.0024	0.0003
10^{-2}	0.1919	0.1159	0.0031	0.0025	0.0004
10^{-3}	0.2371	0.1597	0.0047	0.0028	0.0009
10^{-4}	0.2728	0.2128	0.0062	0.0046	0.0012

Table 2. Results for Case L with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
210	0.0849	0.0489	0.0025	0.0012	0.0004
310	0.1192	0.0712	0.0021	0.0028	0.0004
410	0.1549	0.0928	0.0012	0.0009	0.0005
510	0.1919	0.1159	0.0031	0.0025	0.0005
610	0.2340	0.1368	0.0050	0.0028	0.0005
710	0.2716	0.1590	0.0047	0.0028	0.0003
810	0.3100	0.1834	0.0044	0.0035	0.0004
910	0.3487	0.2056	0.0050	0.0062	0.0009
1010	0.3872	0.2287	0.0072	0.0042	0.0010

Table 3. Results for Case L with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
3	0.1781	0.0988	0.0024	0.0021	0.0004
9	0.1966	0.1130	0.0019	0.0012	0.0004
15	0.1976	0.1156	0.0018	0.0009	0.0001
21	0.2004	0.1157	0.0018	0.0018	0.0003
27	0.1953	0.1140	0.0027	0.0012	0.0007
33	0.1958	0.1153	0.0028	0.0021	0.0007
39	0.1994	0.1174	0.0024	0.0022	0.0004
45	0.2010	0.1175	0.0040	0.0022	0.0005

Table 4. Results for Case QL with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
10^{-1}	0.1665	0.0906	0.0025	0.0021	0.0003
10^{-2}	0.1959	0.1133	0.0028	0.0024	0.0004
10^{-3}	0.2346	0.1603	0.0049	0.0028	0.0006
10^{-4}	0.2762	0.2144	0.0053	0.0041	0.0007

Table 5. Results for Case QL with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
210	0.0814	0.0478	0.0022	0.0006	0.0001
310	0.1155	0.0703	0.0025	0.0012	0.0003
410	0.1593	0.0918	0.0015	0.0027	0.0002
510	0.1959	0.1133	0.0028	0.0024	0.0003
610	0.2331	0.1379	0.0031	0.0021	0.0004
710	0.2736	0.1598	0.0040	0.0015	0.0005
810	0.3115	0.1855	0.0055	0.0031	0.0004
910	0.3522	0.2075	0.0044	0.0019	0.0004
1010	0.3904	0.2305	0.0050	0.0028	0.0006

Table 6. Results for Case QL with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (DMLS)	T_ε: (DMLAS)	T_ε: (SDM)
3	0.1753	0.0984	0.0056	0.0009	0.0001
9	0.1953	0.1153	0.0031	0.0016	0.0003
15	0.1981	0.1187	0.0031	0.0019	0.0002
21	0.1965	0.1196	0.0021	0.0009	0.0002
27	0.1924	0.1141	0.0025	0.0019	0.0001
33	0.1958	0.1163	0.0034	0.0025	0.0003
39	0.1977	0.1162	0.0041	0.0015	0.0001
45	0.1964	0.1158	0.0031	0.0015	0.0006

Table 7. Results for Case Q with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (SQ)	T_ε: (BS)
10^{-1}	0.2815	0.1488	0.0283	0.0762	0.0012	0.0018
10^{-2}	0.3271	0.1929	0.0474	0.1116	0.0015	0.0019
10^{-3}	0.3909	0.2669	0.1049	0.1919	0.0026	0.0040
10^{-4}	0.4575	0.3570	0.1486	0.2534	0.0028	0.0058

Next, in the nonlinear case, we applied (**CGDM**), where (**CGDM0**) denotes the version with zero initial point for any (**CGM**), (**CGDMB**) denotes the version with taking the initial point for any (**CGM**) in the boundary of the feasible set. We utilized these methods with the inexact line search procedure. Therefore, methods (**DML**), (**DMLA**), (**CGDM0**), (**CGDMB**), (**BS**), and (**SQ**) were applied for Case Q. Here (**BS**) and (**SQ**) showed the best performance, and the results of (**CGDM0**) and (**CGDMB**) were better than those of (**DML**) and (**DMLA**).

Table 8. Results for Case Q with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (SQ)	T_ε: (BS)
210	0.1377	0.0833	0.0113	0.0367	0.0006	0.0009
310	0.2004	0.1179	0.0208	0.0580	0.0012	0.0019
410	0.2657	0.1564	0.0321	0.0828	0.0009	0.0018
510	0.3271	0.1929	0.0474	0.1116	0.0015	0.0019
610	0.3900	0.2302	0.0641	0.1396	0.0027	0.0030
710	0.4540	0.2682	0.0728	0.1568	0.0019	0.0034
810	0.5181	0.3052	0.0862	0.1804	0.0022	0.0028
910	0.5809	0.3437	0.0934	0.1960	0.0028	0.0043
1010	0.6434	0.3793	0.1021	0.2110	0.0025	0.0046

Table 9. Results for Case Q with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (SQ)	T_ε: (BS)
3	0.3028	0.1663	0.0465	0.0937	0.0003	0.0031
9	0.3203	0.1881	0.0546	0.0986	0.0015	0.0021
15	0.3250	0.1956	0.0668	0.1205	0.0012	0.0019
21	0.3278	0.1952	0.0541	0.1131	0.0015	0.0022
27	0.3271	0.1957	0.0462	0.1075	0.0012	0.0028
33	0.3300	0.1946	0.0462	0.1111	0.0006	0.0034
39	0.3353	0.1972	0.0365	0.1037	0.0003	0.0028
45	0.3337	0.1974	0.0333	0.0968	0.0006	0.0021

Methods (**DML**), (**DMLA**), (**CGDM0**), (**CGDMB**), and (**BS**) were applied for cases EQ, E, and LG. Here (**BS**) showed the essentially better results than the other methods. Also, (**DMLA**) showed better performance than (**DML**), (**CGDM0**), and (**CGDMB**) in most test experiments.

Table 10. Results for Case EQ with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
10^{-1}	0.2781	0.1478	0.0833	0.3225	0.0040
10^{-2}	0.3281	0.1938	0.1497	0.4153	0.0043
10^{-3}	0.3913	0.2666	0.2547	0.6068	0.0088
10^{-4}	0.4534	0.3572	0.3596	0.7553	0.0103

Table 11. Results for Case EQ with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
210	0.1382	0.0845	0.0293	0.0824	0.0030
310	0.2028	0.1187	0.0674	0.1753	0.0018
410	0.2644	0.1555	0.1127	0.2857	0.0021
510	0.3281	0.1938	0.1497	0.4153	0.0043
610	0.3906	0.2294	0.1693	0.5499	0.0053
710	0.4510	0.2669	0.2031	0.6742	0.0050
810	0.5161	0.3075	0.2312	0.7656	0.0059
910	0.5766	0.3449	0.2718	0.8646	0.0097
1010	0.6421	0.3819	0.3184	0.9579	0.0085

Table 12. Results for Case EQ with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
3	0.3035	0.1671	0.0762	0.3703	0.0040
9	0.3188	0.1871	0.0746	0.3753	0.0030
15	0.3231	0.1932	0.1228	0.4345	0.0046
21	0.3278	0.1949	0.1459	0.4269	0.0053
27	0.3278	0.1924	0.1193	0.3479	0.0047
33	0.3309	0.1955	0.1215	0.3411	0.0041
39	0.3303	0.1981	0.1124	0.2996	0.0053
45	0.3325	0.1949	0.1166	0.2928	0.0052

Table 13. Results for Case E with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
10^{-1}	0.2787	0.1468	1.9862	2.3146	0.0071
10^{-2}	0.3190	0.1884	3.2766	3.6343	0.0083
10^{-3}	0.3887	0.2642	5.1906	5.6561	0.0146
10^{-4}	0.4446	0.3522	6.9235	7.4660	0.0175

Table 14. Results for Case E with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
210	0.1334	0.0797	0.9962	1.0643	0.0028
310	0.1966	0.1143	1.6407	1.7865	0.0078
410	0.2559	0.1506	2.3843	2.6225	0.0081
510	0.3190	0.1884	3.2766	3.6343	0.0083
610	0.3793	0.2231	4.1481	4.6926	0.0090
710	0.4399	0.2612	4.7769	5.4312	0.0136
810	0.5057	0.2981	5.6989	6.5140	0.0140
910	0.5659	0.3353	6.2093	7.1038	0.0156
1010	0.6309	0.3715	7.0919	8.0395	0.0168

Table 15. Results for Case E with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
3	0.2981	0.1662	2.4191	2.8496	0.0072
9	0.3128	0.1828	2.8231	3.2803	0.0068
15	0.3203	0.1896	3.6925	4.1387	0.0106
21	0.3216	0.1918	3.6565	4.0279	0.0103
27	0.3225	0.1881	3.2027	3.5053	0.0089
33	0.3247	0.1907	1.7065	1.9672	0.0109
39	0.3278	0.1916	2.9506	3.2169	0.0099
45	0.3264	0.1906	2.9452	3.1939	0.0113

Table 16. Results for Case LG with $J = 510$, $m = 25$.

ε_λ	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
10^{-1}	0.3617	0.1921	0.1438	0.2213	0.0028
10^{-2}	0.4240	0.2528	0.2318	0.3150	0.0046
10^{-3}	0.5090	0.3452	0.3934	0.5133	0.0051
10^{-4}	0.5900	0.4627	0.5259	0.6784	0.0053

Table 17. Results for Case LG with $m = 25$, $\varepsilon = 10^{-2}$.

J	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
210	0.1780	0.1054	0.0427	0.0703	0.0016
310	0.2585	0.1533	0.0811	0.1240	0.0019
410	0.3391	0.2009	0.1565	0.2247	0.0034
510	0.4240	0.2528	0.2318	0.3150	0.0046
610	0.5109	0.3013	0.3136	0.4281	0.0041
710	0.5962	0.3527	0.4190	0.5507	0.0043
810	0.6819	0.4019	0.5087	0.6494	0.0046
910	0.7675	0.4546	0.6041	0.7487	0.0046
1010	0.8540	0.5038	0.7293	0.8880	0.0064

Table 18. Results for Case LG with $J = 510$, $\varepsilon = 10^{-2}$.

m	T_ε: (DML)	T_ε: (DMLA)	T_ε: (CGDM0)	T_ε: (CGDMB)	T_ε: (BS)
3	0.3975	0.2203	0.6333	0.5357	0.0012
9	0.4262	0.2493	0.4346	0.4997	0.0015
15	0.4302	0.2544	0.3605	0.4422	0.0027
21	0.4303	0.2569	0.2607	0.3452	0.0031
27	0.4234	0.2494	0.1863	0.2740	0.0019
33	0.4246	0.2522	0.1391	0.1968	0.0040
39	0.4253	0.2528	0.1515	0.2444	0.0021
45	0.4278	0.2527	0.1375	0.2346	0.0040

In general, all the suggested methods were rather efficient for these classes of problems. However, we should notice that special decomposable versions of the same methods which exploits peculiarities of each class appeared more efficient that general iterative methods. In particular, proper decomposition of the problem to a set of one-dimensional problems can enhance the convergence essentially. In fact, (**BS**) showed the best results for the nonlinear problems.

5. Conclusions

We considered a general problem of optimal allocation of a homogeneous resource in a wireless telecommunication network with several levels of service. By using the dual Lagrangian method with respect to the capacity constraint, we suggest to reduce the initial problem to a single-dimensional optimization problem. Calculation of the resulting cost function value leads to independent solutions of optimal allocation problems for each kind of service, which can be solved by simple solution methods. The results of computational experiments confirm the efficiency and applicability of the new methods presented.

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