

# Partially Connected Hybrid Beamforming for Large Antenna Arrays in Multi-User MISO Systems

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**Abstract**—Hybrid beamforming (HBF) is a promising approach to be employed in millimeter-wave massive MIMO systems. In this paper, four HBF algorithms with partially connected radio frequency architecture are proposed for large antenna arrays in multi-user MISO systems. The first two algorithms aim at minimizing the difference between either the fully digital zero forcing (ZF) or maximum ratio transmission (MRT) beamformer, and the hybrid beamformer of each user. The other two algorithms apply either ZF or MRT HBF solution to each subarray. The average sum rate performance of the proposed algorithms are evaluated using a realistic geometry-based stochastic channel model, and compared with digital ZF and MRT approaches. Numerical results demonstrate that the subarray-based ZF algorithm is superior to other proposed hybrid methods in all simulation cases.

**Index Terms**—hybrid analog-digital beamforming, massive MIMO, maximum ratio transmission, zero forcing.

## I. INTRODUCTION

The efficient use of new spectrum acquired from higher cm- and mm-wave frequencies plays a key role in fulfilling the stringent performance requirements set for the coming 5G networks. Due to small antenna sizes at higher frequencies, massive MIMO technology with large antenna arrays has been recognized as a promising approach to provide high array gains via narrow beams to combat severe propagation losses occurring in mm-wave frequencies. Massive MIMO provides spectrally efficient communications with high data rates and reasonable cell sizes through (multi-user) spatial multiplexing and beamforming. In multi-user massive MISO systems, lower complexity transmission algorithms, such as zero forcing (ZF) and maximum ratio transmission (MRT), are of practical interest since they may provide relatively good performance [1]. Implementing massive MIMO using conventional digital beamforming is impractical since one radio frequency (RF) chain is needed per

antenna. This is a costly and power-consuming requirement, mostly due to the power consumption of wideband digital-to-analog converters (DACs) [2]. In this respect, hybrid analog-digital beamforming has been proposed as a more practical solution requiring a significantly reduced number of RF chains [3].

There are two main RF architectures in the hybrid beamforming (HBF) procedure, as depicted in Fig. 1. The first one is a fully connected structure in which each RF chain is connected to all antenna elements [3]–[5]. The other one is a partially connected structure where each RF chain is connected to only a subarray of antennas [3], [6]. Plenty of works have been conducted on HBF with fully and partially connected structures proposing different kinds of algorithms from heuristic ones to optimization-based approaches [4]–[9]. For example, a ZF-based HBF approach was developed for mm-wave massive MIMO communications in [9]. Partially connected RF architecture is a practical choice for a massive MIMO type of implementation since it requires a limited number of lossy connections between RF chains and antenna elements [3], [6]. Two different processing strategies can be considered in the context of partially connected HBF, i.e., full array and subarray-based ones. In a full array-based processing strategy, each data stream is connected to all RF chains. Thus, all subarrays participate in transmitting each stream and full array gain is potentially available. Due to partially connected structure, however, beam directions are interdependent resulting in suboptimal beam design process. Using a subarray-based processing approach, each data stream is connected to only one RF chain, and transmitted by a single subarray. This strategy gives more flexibility for choosing beam directions, but limits the array gains by the number of antenna elements in each subarray [10], [11].

In the literature, the analog step of the HBF procedure is conventionally carried out using only phase shifters (PSs). There are various works assuming different configurations and resolutions for PSs [3]–[12]. To improve the flexibility of a HBF procedure, variable gain ampli-

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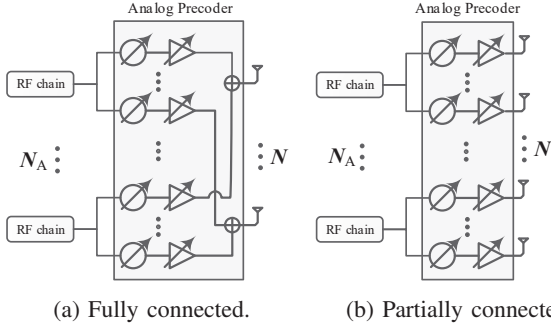


Fig. 1: Analog beamforming network of HBF structures.

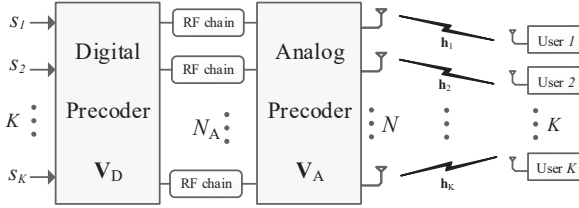


Fig. 2: System model.

fiers (VGAs) can be leveraged in the analog domain as well [13], [14]. Due to the lack of works in the literature, further studies are required for partially connected HBF that employs both analog PSs and VGAs. A performance comparison between full array and subarray-based HBF strategies is of particular interest.

In this paper, four partially connected HBF algorithms are proposed for MU-MISO systems with large antenna arrays. The first two algorithms, ZF-matching and MRT-matching, are designed for the full array processing structure. The other two are subarray-based ZF and MRT algorithms which apply ZF and MRT HBF solutions to each subarray, respectively. The analog beamforming part of the HBF architecture is considered to employ both PSs and VGAs. The average sum rate performance of the proposed HBF algorithms are compared with fully digital ZF and MRT methods. The channel model used for performance evaluation is a realistic geometry-based stochastic approach known as Quasi Deterministic Radio channel Generator (QuaDRiGa).

The remainder of the paper is organized as follows. Section II describes the employed system model and problem formulation. In Section III, the proposed HBF algorithms for MU-MISO systems are presented. Then, the simulation results are investigated and discussed in Section IV. Finally, Section V presents the conclusion of the paper.

## II. SYSTEM MODEL

This section introduces a narrowband single-cell MU-MISO system with partially connected HBF structure operating in a downlink (DL) mode. This system consists

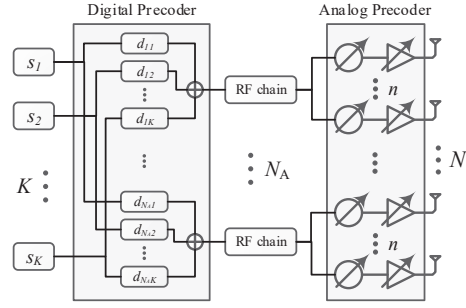


Fig. 3: Partially connected full array-based structure.

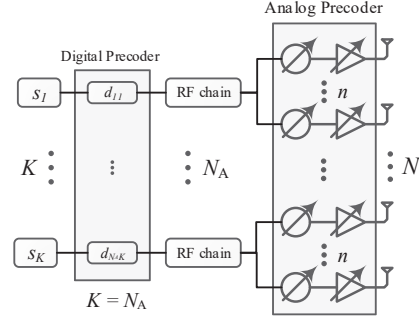


Fig. 4: Partially connected subarray-based structure.

of a base station (BS) with  $N$  antenna elements serving  $K$  single antenna users. It is assumed that perfect channel state information (CSI) is available at the BS which uses a hybrid structure with  $N_A$  RF chains. The HBF structure of the BS is able to apply both amplitude and phase control in the analog part. The antenna array of the BS is divided into  $N_A$  subarrays, each with  $n = N/N_A$  antenna elements. The number of RF chains is considered to be equal to the number of users, i.e.,  $N_A = K$

The MU-MISO system model with HBF at the BS is depicted in Fig. 2. As illustrated in Fig. 3 and Fig. 4, partially connected HBF structure can be divided into full array- and subarray-based processing strategies. In the case of full array-based processing, where all data streams are conveyed to all RF chains, the digital precoder  $\mathbf{V}_D \in \mathbb{C}^{N_A \times K}$  can be expressed as

$$\mathbf{V}_D = \begin{pmatrix} d_{11} & \dots & d_{1K} \\ \vdots & \ddots & \vdots \\ d_{N_A 1} & \dots & d_{N_A K} \end{pmatrix}. \quad (1)$$

In the case of subarray-based processing, each data stream is conveyed to only one RF chain. Hence, the digital precoder becomes a diagonal matrix as  $\mathbf{V}_D = \text{diag}(d_{11}, d_{22}, \dots, d_{N_A K})$ . The analog precoder  $\mathbf{V}_A \in$

$\mathbb{C}^{N \times N_A}$  can be written as

$$\mathbf{V}_A = \begin{pmatrix} \mathbf{a}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{a}_{N_A} \end{pmatrix} \quad (2)$$

where  $\mathbf{a}_i \in \mathbb{C}^{n \times 1}$ ,  $i = 1, 2, \dots, N_A$ , is the analog precoder of the  $i$ th subarray, and  $\mathbf{0} \in \mathbb{C}^{n \times 1}$  is a zero vector. The received signal of the  $k$ th user can be modeled as

$$y_k = \mathbf{h}_k^H \mathbf{V}_A \mathbf{v}_D^k s_k + \mathbf{h}_k^H \sum_{i=1, i \neq k}^K \mathbf{V}_A \mathbf{v}_D^i s_i + z_k \quad (3)$$

where  $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$  is the channel vector from the transmitter to the  $k$ th user,  $\mathbf{v}_D^k \in \mathbb{C}^{N_A \times 1}$  is the  $k$ th column of the digital beamforming matrix,  $s_k$  is the data symbol of the user  $k$ , and  $z_k \sim \mathcal{CN}(0, N_0)$  is the additive white Gaussian noise of the  $k$ th user. The rate expression for user  $k$  can be written as [4]

$$R_k = \log_2 \left( 1 + \frac{|\mathbf{h}_k^H \mathbf{V}_A \mathbf{v}_D^k|^2}{N_0 + \sum_{i=1, i \neq k}^K |\mathbf{h}_k^H \mathbf{V}_A \mathbf{v}_D^i|^2} \right). \quad (4)$$

A common system optimization target is to maximize the average sum rate of all users. The mathematical formulation of the corresponding optimization problem is given by

$$\begin{aligned} & \underset{\mathbf{V}_A, \mathbf{V}_D}{\text{maximize}} \sum_{k=1}^K R_k(\mathbf{V}_A, \mathbf{V}_D) \\ & \text{s.t.} \quad \text{tr}(\mathbf{V}_A \mathbf{V}_D \mathbf{V}_D^H \mathbf{V}_A^H) \leq P \end{aligned} \quad (5)$$

where the optimization variables are the digital beamformer  $\mathbf{V}_D$  and the analog beamformer  $\mathbf{V}_A$  and  $P$  is the maximum available power at the BS. Due to its non-convexity, problem (5) cannot be optimally solved as such. Convex approximation methods can be applied to find sub-optimal solutions. Due to their high computational complexity, the target of such sub-optimal methods is to provide performance upper bounds for lower complexity heuristic methods. When it comes to MIMO systems with large antenna arrays, ZF and MRT beamforming-based methods seem attractive due to their lower complexity and relatively good performance [3], [9].

### III. HYBRID BEAMFORMING ALGORITHMS

In this section, the proposed HBF algorithms for partially connected MU-MISO systems with large antenna arrays are described. The first two algorithms employ full array-based processing structure while the last two algorithms use subarray-based processing approach. For all HBF algorithms, equal power allocation is used, i.e.,  $P_k = P/K$ ,  $k = 1, \dots, K$ . Power values are stacked in a diagonal matrix as  $\mathbf{P} = \text{diag}(P_1, P_2, \dots, P_K)$ .

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#### Algorithm 1 ZF-matching algorithm

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- 1: Initialize  $\mathbf{V}_A$
  - 2: **repeat**
  - 3:   Solve  $\mathbf{V}_D$  from (7) while  $\mathbf{V}_A$  is fixed
  - 4:   Solve  $\mathbf{V}_A$  from (8) while  $\mathbf{V}_D$  is fixed
  - 5: **until** meeting {Threshold | Max iterations}
  - 6:  $\bar{\mathbf{V}} \leftarrow \text{normalize } \mathbf{V}_A \mathbf{V}_D$
  - 7:  $\mathbf{V} \leftarrow \sqrt{\mathbf{P}} \bar{\mathbf{V}}$
  - 8: **return**  $\mathbf{V}$
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#### A. Full array-based Processing

1) *ZF-matching*: This algorithm intends to minimize the difference between the fully digital ZF beamformer and the hybrid beamformer of each user. The corresponding optimization problem can be expressed as

$$\underset{\mathbf{V}_A, \mathbf{V}_D}{\text{minimize}} \|\mathbf{V}_{ZF} - \mathbf{V}_A \mathbf{V}_D\|_F \quad (6)$$

where  $\mathbf{V}_{ZF}$  is the fully digital beamformer containing the ZF beamforming vectors of all users. Problem (6) is non-convex with respect to  $\mathbf{V}_A$  and  $\mathbf{V}_D$ . Hence, it cannot be optimally solved as such. However, a sub-optimal solution can be found by using an alternating optimization strategy where one variable is optimized while the other variable is fixed. The optimization process begins with initializing the analog beamformer and optimizing the digital beamformer as

$$\underset{\mathbf{V}_D}{\text{minimize}} \|\mathbf{V}_{ZF} - \mathbf{V}_A \mathbf{V}_D\|_F. \quad (7)$$

Then, fixing the digital optimized beamformer, the analog beamformer can be solved from

$$\underset{\mathbf{V}_A}{\text{minimize}} \|\mathbf{V}_{ZF} - \mathbf{V}_A \mathbf{V}_D\|_F. \quad (8)$$

This procedure is repeated until either the difference between two subsequent iterations passes a threshold or the maximum number of iterations is reached. Convergence to a sub-optimal solution is guaranteed since each optimization step is convex and improves the objective value. These optimization problems can be solved using standard optimization tools. Note that an orthogonality constraint can be added to the digital beamforming optimization step in (7) in order to make hybrid beams orthogonal. The constraint can be written as

$$(\mathbf{V}_A \mathbf{V}_D)_i^H (\mathbf{V}_D \mathbf{V}_A)_j \leq \epsilon \quad \forall i \neq j. \quad (9)$$

Finally, the overall hybrid precoder is formed as  $\mathbf{V} = \sqrt{\mathbf{P}} \bar{\mathbf{V}}$  where  $\bar{\mathbf{V}}$  is the normalized precoder. The ZF-matching method is summarized in Algorithm 1.

2) *MRT-matching*: This algorithm aims at minimizing the difference between the fully digital MRT beamformer and the hybrid beamformer of each user. The optimization process follows similar steps as in the ZF-matching

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**Algorithm 2** MRT-Matching algorithm
 

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- 1: Initialize  $\mathbf{V}_A$
  - 2: **repeat**
  - 3:   Solve  $\mathbf{V}_D$  from (11) while  $\mathbf{V}_A$  is fixed
  - 4:   Solve  $\mathbf{V}_A$  from (12) while  $\mathbf{V}_D$  is fixed
  - 5: **until** meeting {Threshold | Max iterations}
  - 6:  $\bar{\mathbf{V}} \leftarrow$  normalize  $\mathbf{V}_A \mathbf{V}_D$
  - 7:  $\mathbf{V} \leftarrow \sqrt{\mathbf{P}} \bar{\mathbf{V}}$
  - 8: **return**  $\mathbf{V}$
- 

case. The MRT-matching optimization problem can be formulated as

$$\underset{\mathbf{V}_A, \mathbf{V}_D}{\text{minimize}} \|\mathbf{V}_{\text{MRT}} - \mathbf{V}_A \mathbf{V}_D\|_F \quad (10)$$

where  $\mathbf{V}_{\text{MRT}}$  is the fully digital beamformer containing the MRT beamforming vectors of all users. Due to the non-convexity of (10), alternating optimization strategy is applied to find a sub-optimal solution. The optimization procedure is started by initializing the analog beamformer and optimizing the digital beamformer as

$$\underset{\mathbf{V}_D}{\text{minimize}} \|\mathbf{V}_{\text{MRT}} - \mathbf{V}_A \mathbf{V}_D\|_F. \quad (11)$$

Then the resulting optimized digital beamformer is fixed and the optimization process is done for the analog beamformer as

$$\underset{\mathbf{V}_A}{\text{minimize}} \|\mathbf{V}_{\text{MRT}} - \mathbf{V}_A \mathbf{V}_D\|_F. \quad (12)$$

This process is carried out until a desired level of convergence is achieved. The hybrid precoder is formed as  $\mathbf{V} = \sqrt{\mathbf{P}} \bar{\mathbf{V}}$ . The MRT-matching approach is summarized in Algorithm 2.

### B. Subarray-based Processing

In the subarray-based processing method, each subarray is dedicated to one user. This implies that each data stream is related only to one RF chain and consequently to only one subarray. Hence, the digital precoder becomes a diagonal matrix as  $\mathbf{V}_D = \text{diag}(d_{11}, d_{22}, \dots, d_{KK})$ , just to route the streams to RF chains and the beamforming is carried out in the analog domain.

1) *Subarray-Based ZF*: In the subarray-based ZF method, each subarray uses ZF beamformer to its corresponding user while nulling the interference to the other users. To cancel the interference successfully in this method the number of antennas of each subarray is required to be more than or equal to the number of users. Stacking the channel vectors, the normalized subarray-based ZF beamformer can be formed as

$$\bar{\mathbf{V}} = \begin{pmatrix} \frac{[\mathbf{V}_1]_1}{\|[\mathbf{V}_1]_1\|} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \frac{[\mathbf{V}_2]_2}{\|[\mathbf{V}_2]_2\|} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \frac{[\mathbf{V}_K]_K}{\|[\mathbf{V}_K]_K\|} \end{pmatrix} \quad (13)$$

where  $\mathbf{V}_k = \mathbf{H}_k^H (\mathbf{H}_k \mathbf{H}_k^H)^{-1}$ . The matrix  $\mathbf{H}_k \in \mathbb{C}^{n \times K}$  is the channel between  $k$ th subarray and all  $K$  users, and  $[\cdot]_k$  denotes the  $k$ th column of its matrix argument. Then the overall beamformer is formed as  $\mathbf{V} = \sqrt{\mathbf{P}} \bar{\mathbf{V}}$ .

2) *Subarray-Based MRT*: In this method, each subarray applies MRT solution to its corresponding user. The normalized subarray-based MRT beamformer can be obtained from

$$\bar{\mathbf{V}} = \begin{pmatrix} \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{h}_2}{\|\mathbf{h}_2\|} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \frac{\mathbf{h}_K}{\|\mathbf{h}_K\|} \end{pmatrix} \quad (14)$$

where  $\mathbf{h}_k$  denotes channel vector from the  $k$ th subarray to its corresponding user. Then the overall beamformer is formed as  $\mathbf{V} = \sqrt{\mathbf{P}} \bar{\mathbf{V}}$ .

## IV. SIMULATION RESULTS

In this section, the performance of the proposed HBF algorithms are studied and compared with fully digital ZF and MRT methods. Performance is measured in terms of sum rate against cell size, and the number of users and antennas. The simulation model consists of a single-cell MU-MISO setting with a BS serving  $K$  single antenna users in DL. The HBF structure is assumed to be partially connected with  $N_A = K$ . The employed channel model to generate the MU-MISO channel realizations is Quasi Deterministic Radio channel Generator (QuaDRiGa) [15]. The channel parameters are determined stochastically, based on statistical distributions that are extracted from channel measurements [15]. The measurements used in this paper were conducted at 10 GHz by Centre for Wireless Communications (CWC) at the campus of the University of Oulu, Oulu, Finland [16].

The cell radius  $R$  is considered to be the distance from BS in which all users are distributed randomly in each channel realization. Moreover, indoor LOS propagation environment, including realistic path loss model is chosen. The valid range for path loss in CWC measurements is from 3 to 20 meters. The considered path loss model is formulated as [16]

$$\text{PL} = 10\alpha \log_{10} \left( \frac{d}{d_0} \right) + \beta \quad (15)$$

where  $d$  and  $d_0 = 1$  m denote the distance between the transmitter and receiver and the reference distance, respectively.  $\alpha$  is the coefficient showing the distance dependence, and  $\beta$  is the path loss at the reference distance of 1 m. The main simulation settings are presented in Table I.

Fig. 5 shows average sum rate performance against cell radius with  $N = 64$  t,  $N_A = 4$ , and  $K = 4$ . As it can be seen, ZF-based methods significantly outperform their MRT counterparts. This is due to the fact that SNRs are

TABLE I: Simulation parameters

Channel model	QuaDRiGa
Center frequency	10.1 GHz
Total transmission power	12 W
Noise power	$2.02 \times 10^{-11}$ W
$\alpha$	1.4
$\beta$	56.5
Cell radius	4-20 m
Number of users	2-10
Number of transmit antennas	16-128

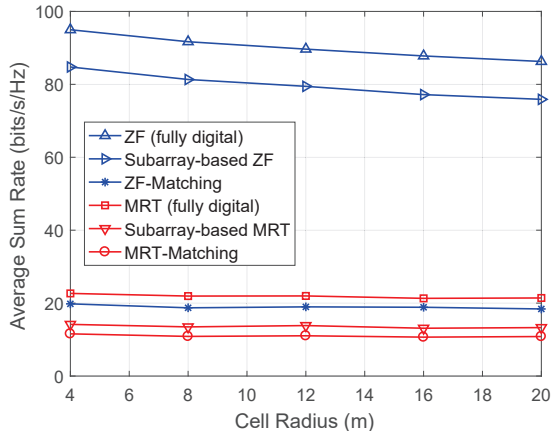


Fig. 5: Average sum rate vs. cell radius with  $N = 64$  and  $N_A = K = 4$ .

high in the chosen indoor scenario with relatively small cell sizes. Thus, MRT suffers from high interference while ZF can null it out. It seems that an array of 64 antennas is not large enough to alleviate the harsh interference conditions for MRT. The results also demonstrate that the subarray-based ZF method is superior to other hybrid algorithms. Due to limited array gain, there is a moderate performance gap when compared with the fully digital ZF method.

Fig. 6 illustrates average sum rate versus the number of users with  $N = 120$  and  $R = 10$  m. In general, increasing the number of users while other parameters are fixed improves the sum rate performance due to the increase in the number of independent data streams. However, ZF- and MRT-matching algorithms present almost a constant performance. This is because the matching is not ideal and these approaches are not able to null the increasing interference resulted from increasing the number of users. The subarray-based ZF algorithm is still the best among the proposed hybrid methods. However, the performance gap against digital ZF grows with increasing the number of users due to the limited beamforming gain.

Fig. 7 presents average sum rate performance against the number of transmit antennas with  $K = 4$  and  $R = 10$  m. This figure shows that increasing the number

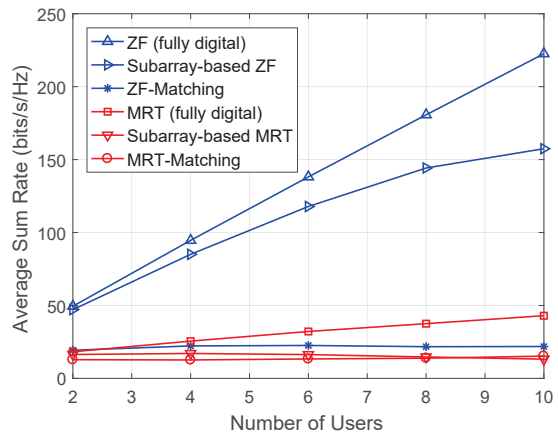


Fig. 6: Average sum rate vs. number of users with  $R = 10$  m,  $N = 120$ , and  $N_A = K$ .

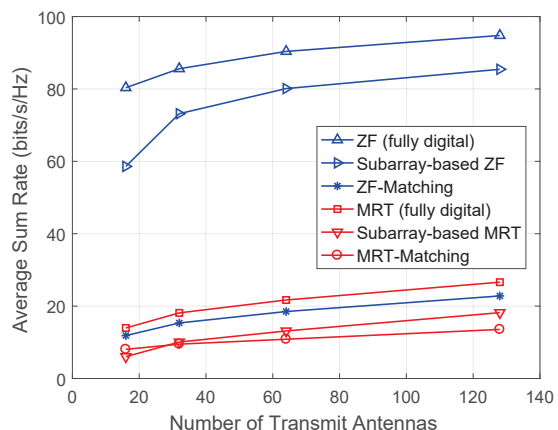


Fig. 7: Average sum rate vs. number of transmit antennas with  $R = 10$  m and  $N_A = K = 4$ .

of transmit antennas improves the sum rate performance of all algorithms due to the enhanced beamforming gain. Again, the digital and hybrid ZF methods notably outperform others.

## V. CONCLUSION

This paper proposed four HBF algorithms for MU-MISO systems employing large antenna arrays and partially connected RF architecture. Algorithms based on ZF and MRT principles were developed for full array and subarray-based processing strategies. The average sum rate performance of the proposed HBF methods was compared with fully digital ZF and MRT solutions in a geometry-based stochastic channel model. The simulations showed that the ZF-based methods outperform their MRT counterparts in the considered indoor channel scenarios. Moreover, the subarray-based ZF algorithm was superior among HBF schemes. In comparison with digital beamforming, the results imply that partially connected HBF provide relatively good performance with significantly reduced hardware complexity. Further studies are still required to find proper hybrid algorithms

for practical massive MIMO communications operating in different cm-wave and mm-wave frequencies.

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